AUC Undergraduate Journal of Liberal Arts & Sciences

Capstone Issue Vol. 19 2022

Amsterdam university college
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Published by
inprint.

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The Amsterdam University College (AUC) Undergraduate Journal of Liberal Arts and Sciences is a biannual, interdisciplinary publication showcasing outstanding undergraduate academic papers. The journal aims to demonstrate the strength of undergraduate scholarship at AUC, reflect the intellectual diversity of its academic programme, and encourage the development of research and writing skills. As an AUC committee, the editorial board of InPrint strives to facilitate collaboration between students and faculty across the curriculum and to provide students with opportunities to gain experience in academic reviewing, editing, and publishing.

Preface by the Dean

Capstones are described in AUC's guidelines as 'the culmination of skills, knowledge, and attitudes students have acquired in their curriculum'. Some of the important requirements for successful completion of a research project rely on the personal attributes of the individual researcher, such as patience, curiosity, openness, creativity, and perseverance. Others concern the wider context within which the research has been conducted. This includes, among others, the existing body of knowledge, the availability of relevant data, and the help and feedback received from others.

The official output of a research project cannot possibly convey all these aspects. It can only give an impression of the effort, enthusiasm, and dedication with which the results were obtained. This applies equally to the six capstones that are featured in this issue, the topics of which range from the way infections may interact with each other, to how art can inform us about the relation between a country's identity and its past. All of the projects raise and address important questions, draw on different perspectives and approaches, and yield results the relevance of which is beyond doubt. For these reasons, I can proudly say that the research featured here is an excellent representation of the breadth, variety, and importance of AUC's Liberal Arts and Sciences programme.

Prof. dr. M. van Hees, Dean AUC

Introduction by the Editor

I am incredibly proud to introduce the 19th Volume of Amsterdam University College's *Undergraduate Journal of Liberal Arts and Sciences*. The theme of this year's edition is: Interdisciplinary perspectives on global challenges.

Understanding our world is the key to engaging with it. However, with increasing complexity, we need more and more perspectives. Therefore, this issue casts a light on young scholars who relate from their particular position in Amsterdam to the world's challenges. Their works combine diverse methods into novel approaches, explain difficult concepts in an accessible way, and acknowledge the complexity required to address these topics.

At the end of their undergraduate studies at AUC, every student conducts an independent research project, which culminates in a capstone, a written thesis that analyzes the academic landscape in a given field, identifies a research need, and uses an innovative methodology to answer the research question. The capstones in this issue excel in all of these aspects, highlighting the potential of undergraduate scholarship. Spanning from discussions about visuals in mathematics to slavery memorials in the Netherlands, from an acclaimed art exhibition in Poland to a suburb in South Africa, these articles reflect the intellectual diversity of the *Liberal Arts and Sciences*.

In the first article, Sarah Martinson uses a complex system approach to model the interactions between COVID-19, TB, and HIV. Since isolated experiments on a single infection can only provide limited insights on real transmissions, Sarah's model derives valuable new insights and policy suggestions. Sarah illustrates that mathematics is one of the most powerful tools we have to understand nature. Therefore, being able to grasp mathematical concepts is important to promote analytical thinking. In her Capstone, Yuval Goren argues that the use of visualizations may help to achieve this objective. However, as their use is controversial in the academic community, Yuval combines a historical overview with an empirical study to advocate a clear benefit of using visuals.

In the Social Sciences, Lara Marie Ziener's analysis of the varying robustness of mobile money frameworks in Afghanistan compared to Kenya illustrates the intertwined relationship between economics, sociology, and (geo)politics. She emphasizes that country-specific factors need to be considered to ensure financial inclusion and stability. Embarking from Europe to Argentina, Adriana Scribani uses a very different method to capture how a cultural traumatic past narrates individual identities far from home. Her qualitative approach through in-depth interviews shows how personal stories relate to historical memory. Together, both capstones pay tribute to the methodological variety in the Social Sciences.

Yet, there is no need to travel far when reflecting on global historical issues such as slavery. Thea Blath Hansen thus integrates investigative research and synthetic mapping of current developments to give an overview of the current racial redress trend in the Netherlands concerning slavery memorials. Using established framework for her analysis, she shows how the Humanities can relate history to the present in a meaningful way. Similarly, Melania Miekus' exploration of Polish-Jewish history through art, in particular Wilhelm Sasnal's exhibition *Such a Landscape*, offers yet another lens. As art can express aspects that other, more analytical forms, cannot capture, this final capstone offers a novel way to see the cultural and historical tensions that may underlie many issues in modern Europe.

This journal would not be possible without the passionate and engaging work of everyone at InPrint. In particular, I want to thank Casey, Filip, and Miriam. We embarked on this project six months ago and I could always rely on when things felt uncertain but never impossible because I know I had you on my side. I am proud of all the new editors who have impressed me with their initiative, engagement, and reliability. You are the ones who give the editorial board life! And especially Lena, Martyna, and Alice who are the (administrative) heart of InPrint. I also want to thank Dr. Belinda Stratton and Prof. Dr. Martin van Hees for their support to make InPrint an official AUC committee and for all the valuable conversations and tips along the way. Finally, every student and every teacher at AUC contributes to this issue by enriching this institution with their never-ending ideas, curiosity, and kindness.

Jannik Faierson, on behalf of InPrint

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Amsterdam University College

Sciences

A dynamical systems approach to modelling the interacting TB, HIV, and COVID-19 epidemics

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Abstract

This paper models TB, HIV and COVID-19 co-infection. Examining interactions between these three infections may provide insights into theoretical transmission dynamics and system behaviour to inform how TB/HIV could impact the trajectory of COVID-19, and how interactions may alter their progressions. This paper considers a simplified dynamic compartmental model specifically allowing for co-infection(s). Little mathematical research has focused on these interactions, and any theoretical knowledge gained by these investigations may prove valuable for infection control. We compute independent basic reproduction numbers for TB ($\mathcal{R}_T$) and HIV ($\mathcal{R}_H$) and show that infection-free equilibria in these independent sub-models are globally asymptotically stable for values of the corresponding basic reproduction number less than unity. We then calculate the basic reproduction number for a system containing both infections: $\mathcal{R}_{TH} = \max\{\mathcal{R}_T, \mathcal{R}_H\}$. This sub-model is applied to Soweto, a region in Gauteng, South Africa, with a high burden of TB and HIV. Numerical simulations reveal that, when the basic reproduction number is less than unity, the infection-free state of the system appears to be stable. The full model containing all three infections is then applied to the same region and the infection-free state and endemic equilibria are examined. The infection-free state for the full system is stable in the simulations. However, when all three independent basic reproduction numbers are greater than unity, the system finds an endemic equilibrium for TB and COVID-19, but HIV is eradicated. The net effect of COVID-19 stabilises in the population; thus, we use the total proportion of COVID-19 cases to examine dependence on $\mathcal{R}_C$, COVID-19's independent reproduction number, in the phase space. When $\mathcal{R}_C < 1$, COVID-19 is eradicated from the system, but as $\mathcal{R}_C$ increases, COVID-19 stabilises with increasing proportions of the population infected. Although the model is simplified and the projection of HIV eradication is unlikely, the qualitative analysis highlights $\mathcal{R}_C$ as a control parameter for COVID-19, implying that interventions targeting $\mathcal{R}_C$ by reducing contact between susceptible and COVID-infected individuals are likely the most effective in slowing the spread of the virus.

Keywords: *dynamical systems, compartmental transmission model, COVID-19, TB, HIV, co-infection, epidemiology*

Suggested citation:
Martinson S. (2022). A dynamical systems approach to modelling the interacting TB, HIV, and COVID-19 epidemics *AUC Undergraduate Journal of Liberal Arts and Sciences*, 19(1) pp. 1-35

imprint.

Abbreviations

AIDS	acquired immunodeficiency syndrome
ART	antiretroviral treatment
CDC	Centers for Disease Control and Prevention
COVID-19	coronavirus disease 2019
HIV	human immunodeficiency virus
IFE	infection-free equilibrium
LTBI	latent tuberculosis infection
NGM	next-generation matrix
ODE	ordinary differential equation
SARS-CoV-2	severe acute respiratory syndrome coronavirus 2
TB	tuberculosis
TBD	tuberculosis disease
WHO	World Health Organisation

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1 Introduction

The global coronavirus disease 2019 (COVID-19) pandemic has significant and unpredictable impacts on health systems worldwide. Financial, infrastructure, and health personnel demands of managing this disease have resulted in considerable compromises to public health service delivery. These include treatment delays for chronic patients, overcrowding of hospitals and limited facilities available for new chronic disease diagnoses. Negative consequences are particularly severe for developing nations with high burdens of other infectious diseases and insufficient or inadequate healthcare facilities [1, 2]. Impacts of pre-existing infections on COVID-19 are unclear, but they may interact with COVID-19 on an immunological level. These interactions may worsen the progression of both COVID-19 and the other pre-existing infections.

Tuberculosis (TB) is the leading global infectious cause of death [3]. Control measures introduced to curb the spread of TB and prevent progression from latent tuberculosis infection (LTBI) to active tuberculosis disease (TBD) have reduced the number of new cases globally [3]. However, the recent COVID-19 pandemic has had negative impacts on TB control programmes [4]. The World Health Organisation (WHO) estimates 21% fewer TB patients received care in 2020 than in 2019 because resources were diverted to COVID-19 responses [5]. Other possible causes of decreased TB notifications include fear of COVID-19 infection at TB testing centres, COVID-19 stigma affecting TB patients with similar symptoms to COVID-19 [6], and restricted access to diagnosis or treatment because of isolation measures. Moreover, lockdowns can increase the number of cases of late or non-diagnosis of TB and heighten exposure of susceptible individuals to infectious, untreated patients in the household, which likely increases risks of LTBI. This can result in increased TBD fatality and transmission within households and communities, as well as more severe disease outcomes [7]. Human immunodeficiency virus (HIV) prevalence is significantly correlated with TB disease burden [8], and consequences are particularly dire in countries with a high HIV burden, as untreated HIV-positive TBD patients experience increased risk of death [9]. Moreover, recent delays in antiretroviral treatment (ART) distribution as a result of lockdown measures put untreated HIV-positive patients at higher risks of progression to severe HIV/acquired immunodeficiency syndrome (AIDS) and TBD [10].

Further, incomplete TB treatment and co-infection with HIV facilitates the emergence of drug-resistant TB strains [11]. Patients with TB have an increased risk of infection with COVID-19 and a predisposition for developing severe COVID-19 symptoms because of a shared dysregulation of immune responses, which may in turn cause TB progression [12, 13]. Moreover, TB lung damage has been associated with severe COVID-19, and high-dose corticosteroids used to treat COVID-19 may initiate progression from LTBI to TBD [14].

However, quarantine and physical distancing measures imposed to prevent spread of COVID-19 may reduce TB transmission [4]. Both TB and severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) are propagated via droplet spread, therefore reduced physical contact likely interrupts transmission of both pathogens. Indeed, a global decline in cases of influenza and respiratory syncytial virus infections was reported, attributable to isolation, mask-wearing and hand-washing measures implemented to combat COVID-19 [15]. Although [16] found it unlikely that TB transmission reductions due to COVID-19 mitigation measures could compensate for reduced testing and other health service disruptions, they possibly have a counterbalancing effect. This could influence future intervention strategies and decisions regarding TB- and HIV-related health services during and after the COVID-19 pandemic.

The triple burden of TB/HIV/COVID-19 is likely to emerge as a major global health challenge. Thus, a thorough theoretical understanding of interactions between these three infections is vital to plan effective intervention methods. Multiple factors must be considered in predicting consequences of co-infection. Lockdowns to contain COVID-19 may have caused a break in the chain of TB management, treatment, and testing, with the likely outcome of worsened prognosis—specifically for undiagnosed patients or those co-infected with HIV. In contrast, these measures may prevent TB transmission. Because the COVID-19 pandemic is ongoing, it is particularly difficult to quantify the combination of these superimposed effects in the short, medium, and long term. Furthermore, these infection interactions are novel, and because TBD progression is relatively slow [11], data describing long-term effects of COVID-19 are not yet available. Mathematical models provide valuable alternatives to raw data analysis, contributing information to assess theoretical possibilities.

Dynamical models are examples of such models that describe changes in quantities over time, incorporating contact between categories of individuals. They can be used to describe risks of transmission of infectious diseases within a population. In such models, the risk of infection depends on the infectious proportion of the population and changes as the number of infectious individuals increases or reduces [17]. Dynamic transmission models enable calculation of the basic reproduction number $\mathcal{R}_0$—the average number of secondary infections produced by a single infectious patient assuming an entirely susceptible population. They also allow for an analysis of infection systems' equilibria—states in which effects of all influences acting within the system are 'cancelled out' by others, resulting in an unchanging, stable state [17]. For example, in the case of infectious diseases, the number of new infections is equal to the number of recoveries/deaths, resulting in an overall zero net change in infections. The infection-free equilibrium (IFE) state at which no infection or disease is

present in a system can be found and analysed [18], providing a target for transmission control. Further, numerical simulation of these systems is possible using parameter values from specific geographic locations. Together, these analyses may inform public health policy and interventions as they provide a broad understanding of infection dynamics from a theoretical, mathematical perspective.

A model and analysis of TB, HIV and COVID-19 individually and in combination is vital to increase our understanding of infection progression and to test the impact of preventive measures in silico. This paper presents a simplified deterministic model using ordinary differential equations (ODEs) describing joint dynamics of TB, HIV and COVID-19. The compartmental model considers co-infections and can be used to examine interactions between these three infections with the aim of providing insights regarding theoretical transmission dynamics. These analyses can increase our understanding of infection interactions and provide mathematical results to aid development of targeted intervention strategies.

An overview of previous models of the three infections is provided in Section 2. Section 3 introduces the system of equations used to model infection interactions, describes model assumptions, and establishes the mathematical well-posedness[1] of solutions on a biologically feasible, positively invariant region. Section 4 analyses the TB-only, HIV-only and TB/HIV (pre-COVID-19) sub-models to gain an understanding of the dynamics of the individual infections. For each subsystem, the basic reproduction number is calculated and the stability of the IFE explored. For the two individual infection sub-models, endemic equilibria are found. Section 5 reports the basic reproduction number for the full model and the results of numeric simulations for the population in Soweto, an urban complex in Gauteng, South Africa, which has a high prevalence of TB and HIV. The IFE of the full system is explored, applying Denmark's low TB and HIV transmission rates to the Soweto model. Although these rates are not applicable to Soweto, which experiences much greater transmission rates, this allows for stability of the IFE to be examined qualitatively. Thereafter, transmission rates for Soweto are modelled to provide more realistic projections for the region. Finally, dependence of the COVID-19 equilibrium on its basic reproduction number is explored. Section 6 provides a summary and discussion of results, including limitations of the current paper and suggestions for future research.

[1] A well-posed problem is one that satisfies three criteria: a solution exists, that solution is unique, and its behaviour changes continuously with the initial conditions of the problem. These properties are commonly required for models describing physical phenomena.

2 Research Context

Compartmental dynamical systems modelling is frequently used to analyse infectious diseases. The population is divided into broad, mutually exclusive subgroups (also called compartments or classes), which vary in number between models depending on important states in the system. The model then tracks individuals in these subgroups. When characteristics of an individual change, they 'move' to the corresponding compartment describing their new state. In deterministic models, this movement is governed by a system of differential equations that considers average population changes. These models are built under simplified mechanistic and phenomenological assumptions and ignore potential effects of chance on the system. Nonetheless, they provide a means of long-term analysis of co-evolving and interacting processes such as infectious diseases, increasing understanding of their possible trajectories, and thereby informing public health policies [11].

Generally, qualitative dynamics of infection models are governed by the basic reproduction number $\mathcal{R}_0$. This dimensionless number describes the expected number of secondary infections caused by a single infectious case in a population where no member is immune and there are no other infectious cases. The basic reproduction number depends on the infectiousness of the microorganism, the duration of infectiousness of cases, and the number of people in the population in contact with the infectious case [19]. It is thus not a biological constant, but dependent on the behaviour of the considered population and is, therefore, model-dependent. On its own, $\mathcal{R}_0$ does not describe the rate of spread of infection in a population, but in most systems it does suggest limiting behaviour [11]. Calculated values for $\mathcal{R}_0$ determine whether an emerging infection will spread in a population and what immunisation coverage is required for disease reduction and eradication. Generally, when $\mathcal{R}_0 < 1$, each case infects on average less than one additional person and new infections will cease, but if $\mathcal{R}_0 > 1$, the infection will spread. Larger $\mathcal{R}_0$ values imply rapidly growing epidemics which are difficult to control. An analytical approach to dynamical systems governing infection transmission can reveal threshold values of $\mathcal{R}_0$ that define a change in the stability of endemic and disease-free equilibria [20], which are significant when considering infection control. Stability implies a closeness or eventual convergence of solutions to a particular state or orbit, thus stable disease-free equilibria are desired.

The literature reports variations of this approach to study disease transmission. Of specific relevance are those considering interaction(s) between some combination of TB, HIV and COVID-19, and those considering co-infection of multiple diseases in a single host.

Dynamics of TB and HIV have been analysed individually (see [21, 22, 23, 24, 25, 26] for TB models and [27, 28, 29, 30] for HIV models). However, for regions with

a high burden of both infections, studying their dynamics separately may not provide accurate results because they are markedly correlated. Several studies have investigated the interacting epidemics by including compartments that allow for infection with either one or both of TB/HIV [8, 31, 32, 33, 34, 35, 36]. These studies compute the basic reproduction numbers (in terms of model parameter values) for HIV and TB independently. Moreover, they propose methods for analysis of equilibria and stability of dynamical models of these infections, indicating that such analyses are possible despite the complicated nature of co-infection. Individual reproduction numbers for HIV and TB are calculated separately, and the maximum of these are taken as the overall reproduction number of the system. In general, if this $\mathcal{R}_0 < 1$, the disease-free equilibrium is locally asymptotically stable [8, 31, 32], but unstable if $\mathcal{R}_0 > 1$ [31].

Similar methods have been used to analyse COVID-19. Although it is relatively novel, much is known about its transmission, allowing for the application of dynamical systems modelling approaches. Examples include the simple Susceptible-Exposed-Infected-Recovered (SEIR) model [37], the more complex Susceptible-Exposed-Asymptomatic-Infected-Recovered-Susceptible (SEAIRS) model [38], and an extended SEIR model describing four different infected compartments, which also models effects of isolation [39]. In all three models, COVID-19's disease-free equilibrium was locally asymptotically stable if $\mathcal{R}_0 < 1$. However, these models analyse COVID-19 in isolation without considering potential effects from co-infections.

Effects of COVID-19 on chronic illnesses and their progression, including effects on the TB epidemic in high-burden areas, have been examined [40]. This involved an adaptation of preexisting dynamic TB transmission models to include potential impacts of lockdown and other preventive measures on TB care in three settings with markedly different TB burdens. The model suggested that slight disruptions in TB care may result in long-term increases in TB incidence and deaths, but that these adverse effects could be mitigated with targeted interventions implemented early after the onset of the pandemic. This approach has been extended in a model of specific effects of multiple COVID-19 scenarios on TB, HIV and malaria. Interruptions in care and possible reductions in transmission due to physical distancing and lockdown measures were modelled [41]. Four possible courses of the COVID-19 pandemic with resulting deaths were compared with the deaths attributable to each of the three other infections in scenarios where prevention and treatment programmes were interrupted in 2020. Despite these extensive investigations into effects of COVID-19 on other infections, a major limitation to these studies is that the models for COVID-19, TB, HIV and malaria were separate, therefore the compounding effects of interactions between the infections were not examined [41]. This suggests that models should be expanded to include co-infections to explore compounding and protective factors for transmission [42].

A general model for two infections, one chronic and one acute, that co-infect a single host, has been developed to include infection interactions [43]. This allowed for an examination of infection dynamics in tandem, finding conditions for stability of their co-existent equilibria. These concepts are applied more specifically by models that allow for acute COVID-19 co-infection interacting with chronic HIV [44] or TB [45]. Both infections are associated with increased risk of COVID-19-associated mortality [46, 47]. HIV and COVID-19 were modelled together in a four-compartment model with rates of transmission weighted by parameters based on the current class [44]. The highest probability of mortality was assigned to the compartment co-infected with both HIV and COVID-19, and a regularisation parameter was used to model relative transmission rates of co-infected patients compared to those infected with HIV only. Stability analyses of sub-models of individual diseases were conducted and used to inform analysis of the full model. The basic reproduction number for the full system $\mathcal{R}_0$ was taken as the maximum of the basic reproduction numbers for COVID-19 and HIV individually. Then, the disease-free equilibrium was found to be stable in the full system for $\mathcal{R}_0 < 1$ but unstable for $\mathcal{R}_0 > 1$ [44]. Interactions between COVID-19 and TB have also been modelled; a stochastic compartmental model was used by [45] to estimate COVID-19 infection and mortality in TB patients in India. This aimed primarily to inform prevention measures; a stability analysis of the system was not performed.

These studies investigated interaction between two infections simultaneously, however, co-infection models can be further extended to encompass interactions of three diseases. TB, HIV and malaria co-infection were modelled deterministically in a single model to gain insight into their combined transmission dynamics [42]. Increased rates of disease progression or transmission due to co-infection were modelled using weighted parameters. A stability analysis was performed, which revealed that effective counselling and treatment for all three infections reduced the value of the basic reproduction number for HIV to below unity, effectively eliminating HIV [42]. This finding specifically highlights the potential tangible public health relevance of dynamical systems modelling, and the importance of modelling joint epidemics. However, a model incorporating direct interactions between TB, HIV and COVID-19 has yet to be developed.

This paper therefore presents a deterministic, dynamic compartmental model governed by ODEs describing the combined TB, HIV and COVID-19 epidemics, including co-infection and changed mortality risks based on infection status. It allows for the evaluation of outcomes resulting from disease competition, interaction or opportunistic infection. An analysis of fixed points in the model and their stability increases theoretical understanding of both those infection interactions that cannot

be ignored and those which may be less important to examine. Interactions between these three infections have not yet been modelled at this paper's time of writing, thus these findings will provide the first theoretical insights into their communal behaviour and will create a platform for the development of informed intervention strategies to mitigate their potential effects.

3 Model Formulation

This model aims to inspect the impacts of interactions between TB, HIV and COVID-19 on their transmission by examining a total population subdivided into 18 mutually exclusive compartments that describe intersections of three TB states (susceptible, LTBI or exposed, TBD), three HIV states (susceptible, HIV-positive, severe HIV or AIDS), and two COVID-19 states (susceptible, infected) as summarised in Table 1[2]. The total population at time t, denoted $N(t)$, is the sum of all compartments at time t.

$$\begin{aligned} N(t) =& S(t) + I_T(t) + D(t) + I_C(t) + I_{TC}(t) + D_C(t) \\ &+ I_H(t) + I_{TH}(t) + I_{HC}(t) + I_{THC}(t) + A(t) \\ &+ I_{TA}(t) + D_A(t) + A_C(t) + I_{TAC}(t) + D_{AC}(t). \end{aligned}$$

Table 1: Model compartments and notation for population groups.

Variable	Description
S	susceptible to all three infections
I_T	LTBI or exposed to TB
D	TBD only
I_C	COVID-19 infected only
I_{TC}	LTBI and COVID-19 infected
D_C	TBD, COVID-19 infected
I_H	HIV-positive only
I_{TH}	LTBI and HIV-positive
I_{HC}	HIV-positive and COVID-19 infected
I_{THC}	LTBI, HIV-positive and COVID-19 infected
A	AIDS only
I_{TA}	LTBI and AIDS
D_A	TBD and AIDS
A_C	AIDS and COVID-19 infected
I_{TAC}	LTBI, AIDS, and COVID-19 infected
D_{AC}	TBD, AIDS, and COVID-19 infected
N	total population

Entry into the susceptible population S is assumed to occur at constant rate b, and all individuals experience a natural death rate μ. Infected and diseased individuals experience a further death rate based on their infection status. In the classical Susceptible-Infected-Recovered (SIR) model, the transition rate from the susceptible compartment S to the infected compartment I is modelled by the 'force of infection' $F = \beta I$, with β the rate of effective (i.e., infection-spreading) contact between individuals in classes S and I per unit time. However, for compartmental models using a large number of classes to describe infectious disease dynamics, it becomes more realistic to consider a force of infection that does not depend on the absolute number of infections, but rather on the fraction of infectious cases in the total population N [31]. This results in a time-dependent force of infection $F(t) = \frac{\beta I(t)}{N(t)}$. Moreover, because this model considers multiple infectious classes for each infection as well as co-infected classes, each pathogen's force of infection is proportional to the sum of all classes describing infectious states of that pathogen.

The time-dependent force of infection $\lambda_T(t)$ associated with TB infection describes the rate at which susceptible individuals acquire LTBI following contact with individuals who have active TBD

$$\lambda_T(t) = \frac{\beta_T}{N}[D + \kappa_{T,H} D_A + \kappa_{T,C} D_C(t) + \kappa_{T,HC} D_{AC}], \quad (1)$$

where β_T is the rate at which susceptible and TB-infectious individuals come into effective contact (sufficient to result in TB infection) per unit time. The modification parameter $\kappa_{T,H}$ accounts for altered TB infectiousness of individuals because of HIV/AIDS co-infection compared to HIV-negative individuals. Similarly, $\kappa_{T,C}$ and $\kappa_{T,HC}$ account for the TB infectiousness of patients with TBD/COVID-19 and TBD/HIV/COVID-19 co-infection, respectively.

Similarly, susceptible individuals acquire HIV infection at rate $\lambda_H(t)$ following effective contact with HIV-positive individuals

$$\begin{aligned} \lambda_H = \frac{\beta_H}{N}[& I_H + A + \kappa_{H,T}(I_{TH} + I_{TA} + D_A) \\ &+ \kappa_{H,C}(I_{HC} + A_C) \\ &+ \kappa_{H,TC}(I_{THC} + I_{TAC} + D_{AC})] \end{aligned} \quad (2)$$

and susceptible individuals acquire COVID-19 infection at rate $\lambda_C(t)$ following effective contact with infectious COVID-19 cases

$$\begin{aligned} \lambda_C(t) = \frac{\beta_C}{N}[& I_C + \kappa_{C,T}(I_{TC} + D_C) \\ &+ \kappa_{C,H}(I_{HC} + A_C) \\ &+ \kappa_{C,TH}(I_{THC} + I_{TAC} + D_{AC})] \end{aligned} \quad (3)$$

As in equation (1), the $\kappa_{i,j}$ parameters in equations (2) and (3) (with $i \in \{H, C\}$ and $j \in \{T, H, C, TH, TC\}$) describe infectiousness of co-infected classes due to co-infection. State i represents the infection whose infectiousness is changed by co-infection with state(s) j.

[2]Note that TBD is an AIDS-defining characteristic, thus it is impossible to have TBD/HIV co-infection without being classified as an AIDS case.

The governing system of ODEs describing interacting transmission dynamics of TB, HIV and COVID-19 is

$$
\begin{cases}
\dot{S} = b - [\mu + \lambda_T + \lambda_H + \lambda_C]S + r_C I_C \\
\dot{I}_T = \lambda_T S - [\mu + f_T + \sigma_{TH}\lambda_H + \sigma_{TC}\lambda_C]I_T + r_C I_{TC} + r_T D \\
\dot{D} = f_T I_T - [(\mu + \mu_T) + \sigma_{TH}\lambda_H + \sigma_{TC}\lambda_C + r_T]D + r_C D_C \\
\dot{I}_C = \lambda_C S - [(\mu + \mu_C) + r_C + \sigma_{CT}\lambda_T + \sigma_{CH}\lambda_H]I_C \\
\dot{I}_{TC} = \sigma_{TC}\lambda_C I_T + \sigma_{CT}\lambda_T I_C - [(\mu + \mu_{TC}) + r_C + \sigma_{TC} f_T]I_{TC} \\
\dot{D}_C = \sigma_{TC}\lambda_C D - [(\mu + \mu_{TC}) + r_C]D_C + \sigma_{TC} f_T I_{TC} \\
\dot{I}_H = \lambda_H S - [(\mu + \mu_H) + \sigma_{HT}\lambda_T + \sigma_{HC}\lambda_C + f_H]I_H + r_H A + r_C I_{HC} \\
\dot{I}_{TH} = \sigma_{TH}\lambda_H I_T + \sigma_{HT}\lambda_T I_H - [(\mu + \mu_{TH}) + \sigma_{TH,C}\lambda_C + f_T + f_H]I_{TH} + r_H I_{TA} + r_C I_{THC} \\
\dot{I}_{HC} = \sigma_{HC}\lambda_C I_H + \sigma_{CH}\lambda_H I_C - [(\mu + \mu_{HC}) + r_C]I_{HC} \\
\dot{I}_{THC} = \sigma_{TH,C}\lambda_C I_{TH} - [(\mu + \mu_{THC}) + r_C]I_{THC} \\
\dot{A} = f_H I_H - [(\mu + \mu_H) + \sigma_{HT}\lambda_T + \sigma_{HC}\lambda_C + r_H]A + r_C A_C \\
\dot{I}_{TA} = \sigma_{HT}\lambda_T A + f_H I_{TH} - [(\mu + \mu_{TH}) + \sigma_{TH,C}\lambda_C + f_T + r_H]I_{TA} + r_T D_A + r_C I_{TAC} \\
\dot{D}_A = f_T I_{TA} + f_T I_{TH} - [(\mu + \mu_{TH}) + \sigma_{TH,C}\lambda_C + r_T]D_A + +\sigma_{TH}\lambda_H D + r_C D_{AC} \\
\dot{A}_C = \sigma_{HC}\lambda_C A - [(\mu + \mu_{HC}) + r_C]A_C \\
\dot{I}_{TAC} = \sigma_{TH,C}\lambda_C I_{TA} - [(\mu + \mu_{THC}) + r_C]I_{TAC} \\
\dot{D}_{AC} = \sigma_{TH,C}\lambda_C D_A - [(\mu + \mu_{THC}) + r_C]D_{AC}
\end{cases}
\tag{4}
$$

Model (4) describes interactions of the three infections, illustrated in Figure 1. Definitions of model parameters are provided in Table 2.

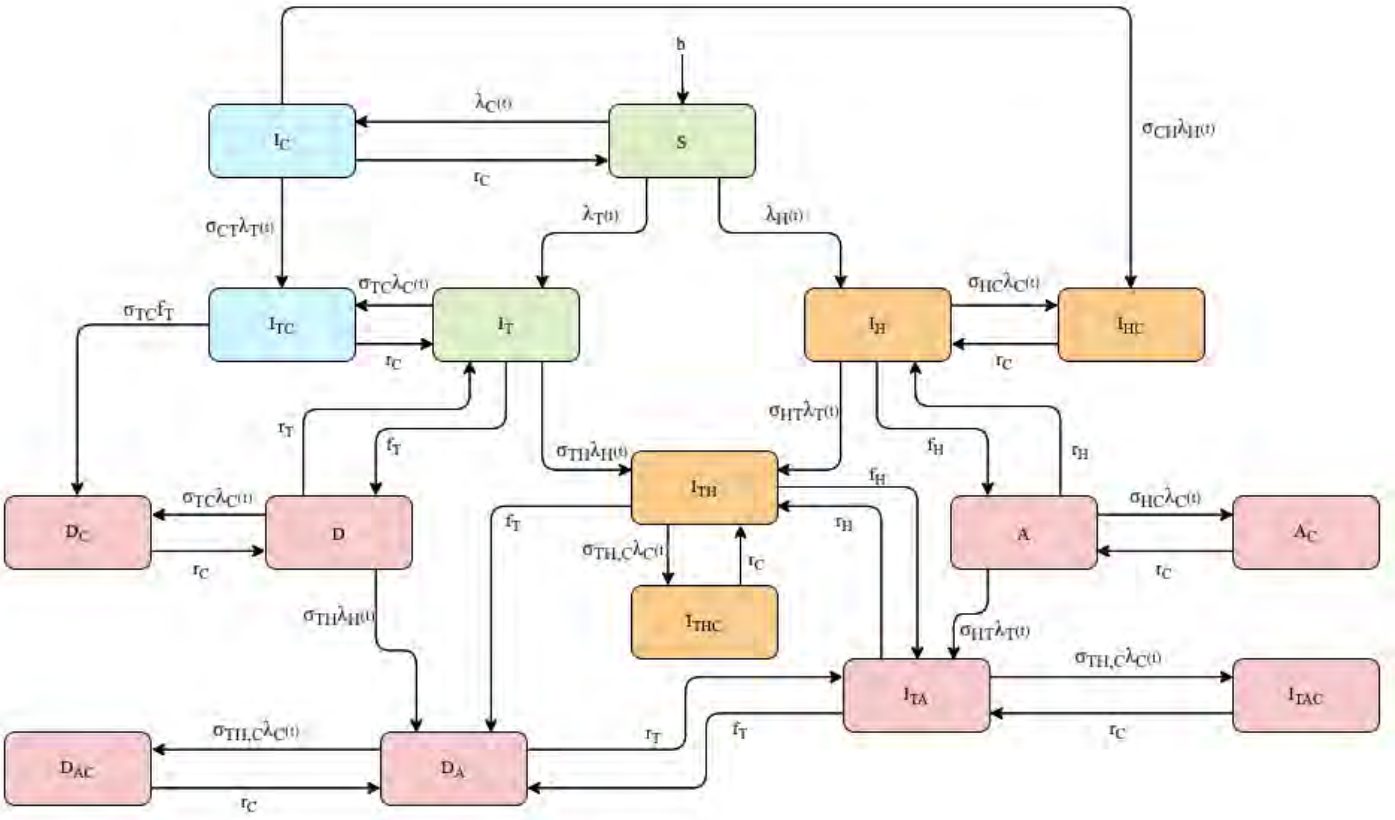


Figure 1: Transition diagram illustrating possible movements between compartments in model (4) and the corresponding average rate at which each motion occurs. All compartments experience natural death rate μ as well as a death rate due to the (co-)infection described by that class. Colours represent infectiousness: non-infectious classes are green, COVID-19-only infectious classes blue, TBD or AIDS-infected class red, and other HIV-infected classes orange.

Table 2: Definitions of parameters used in model (4).

Symbol	Definition
b	constant recruitment rate
μ	per-capita natural death rate
β_T	effective contact rate with individuals with active TBD
f_T	per capita TB progression rate from latent to active
r_T	TBD recovery rate to LTBI
μ_T	per-capita TBD-induced death rate
$\kappa_{T,H}$	TBD infectiousness of individuals also infected with HIV
$\kappa_{T,C}$	TBD infectiousness of individuals also infected with COVID-19
σ_{TH}	HIV susceptibility for individuals with TB
σ_{TC}	COVID-19 susceptibility for individuals with TB
β_H	effective contact rate with HIV-infectious individuals
f_H	per capita AIDS progression rate for HIV-infected individuals
r_H	AIDS recovery rate to HIV
μ_H	per-capita HIV-induced death rate
$\kappa_{H,T}$	HIV infectiousness of individuals also infected with TB
$\kappa_{H,C}$	HIV infectiousness of individuals also infected with COVID-19
σ_{HT}	LTBI susceptibility for individuals with HIV
σ_{TC}	COVID-19 susceptibility for individuals with HIV
β_C	effective contact rate with COVID-19 infectious individuals
r_C	COVID-19 recovery rate
μ_C	per-capita COVID-19 death rate
$\kappa_{C,T}$	COVID-19 infectiousness of individuals also infected with TB
$\kappa_{C,H}$	COVID-19 infectiousness of individuals also infected with HIV
σ_{CT}	LTBI susceptibility for individuals with COVID-19
σ_{CH}	HIV susceptibility for individuals with COVID-19
μ_{TH}	per-capita death rate attributable to TBD/HIV co-infection
μ_{TC}	per-capita death rate attributable to TBD/COVID-19 co-infection
μ_{HC}	per-capita death rate attributable to HIV/COVID-19 co-infection
μ_{THC}	per-capita death rate attributable to TBD/HIV/COVID-19 co-infection
$\kappa_{T,HC}$	TBD infectiousness of individuals also infected with HIV and COVID-19
$\kappa_{H,TC}$	HIV infectiousness of individuals also infected with TB and COVID-19
$\kappa_{C,TH}$	COVID-19 infectiousness of individuals also infected with TB and HIV
$\sigma_{TH,C}$	COVID-19 susceptibility for individuals with TB and HIV

3.1 Assumptions

The model relies on certain assumptions to remain simple to analyse. First, homogeneous mixing in the population is assumed—heterogeneous space is excluded[3]. Second, all members of the population experience homogeneous susceptibility, i.e., they are genetically equally vulnerable to the three diseases. Age, demographic and socioeconomic structures are not included. Third, because COVID-19 is an acute infection while TB and HIV are chronic, assumptions are made about differences in infection and progression possibilities. Prominently, under the current model, most chronic progressions to disease are excluded while an individual is infected with COVID-19. New infections are limited to those with only COVID-19, and not included for COVID-19 co-infections. This is because the infectious period of coronavirus is short (around two weeks), while TB and HIV cases can remain infectious for years. It is unlikely that infections or progressions happen during the relatively short time while someone is co-infected with COVID-19. For example, if someone has had HIV for two years when they are infected with COVID-19, it is improbable that their HIV will progress to AIDS during this two-week window. However, this simplification excludes some potential interactions of co-infection, and assumes that COVID-19 infection does not affect disease progression in the short term as there is little evidence for such altered progression in the existing literature. It is important to note that this assumption also does not exclude interactions entirely, as increased mortality rates due to co-infection are included, and the progression from LTBI to TBD is modelled as increased when an individual is co-infected with COVID-19 because of a likely dual immunosuppression [13]. Fourth, immunisation from COVID-19, either through vaccination or previous infection, is not considered. This decision was made to simplify the model because early in the pandemic there was no vaccine and little information regarding immunity due to previous infection. However, this allows individuals to be continually (re-)infected with COVID-19 immediately after recovery, which is not representative of realistic dynamics. Fifth, it is assumed that TB, HIV and COVID-19 can only be contracted through contact with infectious individuals. Specifically, this means that certain transmission paths of HIV are ignored, including mother-to-child transmissions and shared injection equipment. Finally, sexual transmission of HIV is modelled indirectly (using contact) rather than through specific modes of sexual interaction as this would make the model more difficult to solve analytically. Despite these assumptions and simplifications, the model remains valuable in tracing both general and specific interactions of the three diseases and analysing their dynamics, providing a more holistic understanding.

[3]Essentially, the model makes the assumption that any individual can come into contact with any other individual in the model population, and that these contacts between people are all equally likely.

3.2 Positivity of Boundedness of Solutions

The model system (4) describes biological, living populations, therefore all parameters are non-negative, and associated state variables must also be non-negative for all time $t \geq 0$. Consequently, the solutions of this model with positive initial data remain positive for all time $t \geq 0$ and are bounded in some region. Consider the biologically feasible region

$$\Omega = \Big\{ (S, I_T, D, I_C, I_{TC}, D_C, I_H, I_{TH}, I_{HC}, I_{THC}, A, I_{TA}, D_A, A_C, I_{TAC}, D_{AC}) \in \mathbb{R}^{16}_+ : N \leq \frac{b}{\mu} \Big\}.$$

Theorem 1. *Region Ω is positively invariant with respect to model system (4) with non-negative initial conditions in $\mathbb{R}^{16}_+$. Under these conditions, all solutions for system (4) are positive for $t \geq 0$, and solutions starting in Ω remain in Ω for all time t.*

Proof. The non-zero rate of change of the total population is obtained by summing all equations in model (4):

$$\begin{aligned}\frac{dN}{dt} =& b - \mu N - \mu_T D - \mu_C I_C - \mu_{TC}[I_{TC} + D_C] \\ & - \mu_H[I_H + A] - \mu_{HC}[I_{HC} + A_C] \\ & - \mu_{TH}[I_{TH} + I_{TA} + D_A] \\ & - \mu_{THC}[I_{THC} + I_{TAC} + D_{AC}]\end{aligned}$$

It follows that $\frac{dN}{dt} \leq b - \mu N$. Then, using Gronwall's Lemma [48], since all functions are non-negative

$$0 \leq N(t) \leq N(0)e^{-\mu t} + \frac{b}{\mu}(1 - e^{-\mu t}),$$

where $N(0)$ represents $N(t)$ evaluated at the initial conditions.

As $t \to \infty$, $0 \leq N(t) \leq \frac{b}{\mu}$ and, specifically, if $N(0) \leq \frac{b}{\mu}$ then $N(t) \leq \frac{b}{\mu}$. Therefore, all feasible solutions of model (4) starting in Ω remain there for all time, and Ω is positively invariant. □

It is sufficient to consider the dynamics of the model (4) only in Ω where the system of equations is both mathematically and biologically well-posed, and every solution starting in Ω will remain there for all future time $t > 0$.

4 Sub-model Analysis

Before performing an analysis of the full model with all three of TB, HIV and COVID-19 present, sub-models for TB-only, HIV-only and TB/HIV co-infection will be analysed to provide insight into dynamics of the individual infections.

4.1 TB-only Sub-model

The model incorporating only LTBI and TBD is obtained by setting all states defining HIV, COVID-19 or co-infection to zero: $I_C = I_{TC} = D_C = I_H = I_{TH} = I_{HC} = I_{THC} = A = I_{TA} = D_A = A_C = I_{TAC} = D_{AC} = 0$

$$\begin{cases} \dot{S} = b - [\mu + \lambda_T]S \\ \dot{I}_T = \lambda_T S - [\mu + f_T]I_T + r_T D \\ \dot{D} = f_T I_T - [(\mu + \mu_T) + r_T]D \end{cases} \tag{5}$$

with non-negative initial conditions S_0, I_{T_0}, D_0, and $\lambda_T(t)$ and $N(t)$ defined as

$$\lambda_T(t) = \frac{\beta_T D}{N}$$
$$N(t) = S + I_T + D.$$

The sub-model system (37), which is mathematically and biologically well-posed, will be studied in the biologically feasible region

$$\Omega_T = \left\{ (S, I_T, D) \in \mathbb{R}^3_+ : N \leq \frac{b}{\mu} \right\}. \tag{6}$$

Similarly to Theorem 1 for the full model (4) and based on biological considerations, we can prove that Ω_T is positively invariant.

4.1.1 TB's basic reproduction number

The basic reproduction number $\mathcal{R}_0$ is an important threshold quantity for disease control. It depends on the definition of the infected and uninfected model compartments [19]. If $\mathcal{R}_0 < 1$, an infectious individual produces on average less than one *new* infection during their infectious period and the pathogen cannot spread. However, $\mathcal{R}_0 > 1$, each infected individual infects, on average, more than one other person, and the infection is able to proliferate in the population [20]. Each of the infections in this model has an independent basic reproduction number which refers to the number of secondary infections caused by a single infectious case of that infection in the susceptible population. The basic reproduction number for TB, denoted $\mathcal{R}_T$, is thus defined as the number of secondary TB infections (i.e., LTBI cases) produced by a single case of active TBD. To compute $\mathcal{R}_T$, we distinguish *new* infections with TB from other movements between compartments. Note that all cases in compartment D in this sub-model are *infectious* with active TBD, and no other cases are infectious. However, there are two *infected states*, D and I_T. This means we cannot use the heuristic SIR-model definition of $\mathcal{R}_0$ as the product of the infection rate and the mean infection duration. Instead, we turn to the more general next-generation matrix (NGM) method used to calculate $\mathcal{R}_0$ when a finite number of compartments are included in the model [19, 20]

This method involves the separation of the infected system into two matrices: F containing infection rates and V recovery rates. The product FV^{-1} then represents [infection rates][recovery rates]$^{-1}$; this parallels the heuristic R_0 definition, which can be interpreted as (individual infection rate)(individual recovery rate)$^{-1}$. The product FV^{-1} then essentially represents multiplicative changes in each class from one generation to the next. Approximating the infected classes near the equilibrium and using the eigenvector equation to explore changes in their magnitudes demonstrates that when the absolute value of the corresponding eigenvalue is greater than one, the number of infected individuals increases (i.e., the infection spreads), otherwise the infection is contained. Thus, if all eigenvalues are in $(-1, 1)$, the infection will not spread, but will spread if any eigenvalue lies outside of this region. We can use the dominant eigenvalue[4] to characterise the behaviour of all eigenvalues and thus to determine the possibility of infection spread. The dominant eigenvalue captures same information as R_0, so R_0 is the spectral radius—the largest absolute value of a square matrix's eigenvalues—of the NGM (denoted K) [19][5]. We examine the number of new infections produced by an infectious case in a susceptible population at an IFE or steady-state (a locally asymptotically stable equilibrium solution of the infection-free model).

The *infected subsystem* must be linearised about the IFE where no infection is present. We can take this to represent a point where infection was never introduced and can assume that the IFE is at least stable in this case. This linearisation is not merely theoretical, but has an epidemiological interpretation: it reflects that the basic reproduction number characterises the *potential* for initial infection spread in an entirely susceptible population (assuming a negligible change in the number of susceptible individuals during initial growth of infection).

At the IFE, $I_T = D = 0$, implying $S = N$. Thus, in the linearisation, $\lambda_T = \frac{\beta_T D}{S}$. The linearised subsystem is

$$\begin{cases} \dot{S} = b - \mu S - \beta_T D \\ \dot{I}_T = [\beta_T + r_T]D - [f_T + \mu]I_T \\ \dot{D} = f_T I_T - [(\mu + \mu_T) + r_T]D \end{cases} \tag{7}$$

[4] The real eigenvalue whose absolute value is strictly greater than that of all other eigenvalues.

[5] In this paper, an outline of the method is provided and applied to the current infection model. For a full explanation and for proofs of the method's validity, see [19, 20].

Since this linear system is derived by linearisation, it can be described by a Jacobian matrix with components first order partial derivatives with respect to each state [19]. Instead of examining the Jacobian for the full linearised subsystem (7), we can examine only the *infected* compartments' differential equations $\dot{I}_T$ and $\dot{D}$, excluding $\dot{S}$ and reducing the order of the system. This is because components of the Jacobian $\frac{\partial \dot{I}_T}{\partial S}$ and $\frac{\partial \dot{D}}{\partial S}$ are both zero, and $\frac{\partial \dot{S}}{\partial S} = -\mu$. Thus, the three eigenvalues of the full Jacobian will be $-\mu$ along with the two eigenvalues of the 2×2 *infected* subsystem containing only $\dot{I}_T$ and $\dot{D}$. Since $-\mu < 0$, it is enough to evaluate only the 2×2 *infected* subsystem's eigenvalues. Epidemiologically, this translates as only including states describing new infections or changes between infected states.

The Jacobian for the *infected* subsystem can be decomposed as the sum of two matrices $\mathbf{T}$ and $\mathbf{\Theta}$, where $\mathbf{T}$ describes *transmission* or the production of new infections, and $\mathbf{\Theta}$ describes *transitions* or state changes between infected classes (and deaths). Thus

$$\mathbf{x} = \begin{bmatrix} I_T \\ D \end{bmatrix} \implies \dot{\mathbf{x}} = (\mathbf{T} + \mathbf{\Theta})\mathbf{x} = (\mathbf{T} + \mathbf{\Theta}) \begin{bmatrix} I_T \\ D \end{bmatrix} \tag{8}$$

with $\mathbf{T}$ and $\mathbf{\Theta}$ two-dimensional matrices. It is possible to find $\mathbf{T}$ and $\mathbf{\Theta}$ using a linear algebra approach [20]. However, because $\mathbf{T}$ describes only transmission events, we can find its entries through epidemiological interpretation [19]. Let $i, j \in \{I_T, D\}$ be infected states. Then the matrix entry $\mathbf{T}_{ij}$ represents the rate at which individuals currently in state j generate individuals in state i in the linearised system. Progression from I_T to D and return from D to I_T through 'recovery' are not new infections and are not included.

$$\mathbf{T} = \begin{bmatrix} 0 & \beta_T \\ 0 & 0 \end{bmatrix}. \tag{9}$$

Non-zero rows of $\mathbf{T}$ correspond to *states at infection* (states an individual can be in directly following infection), and non-zero columns correspond to *states of infectiousness*. This formulation of $\mathbf{T}$ confirms that I_T is the only state at infection and D the only state of infectiousness.

Using equations (8) and (9), we can find the transition matrix $\mathbf{\Theta}$ and its inverse

$$\mathbf{\Theta} = \begin{bmatrix} -(f_T + \mu) & r_T \\ f_T & -(\mu + \mu_T + r_T) \end{bmatrix}, \tag{10}$$

$$\mathbf{\Theta^{-1}} = \frac{-1}{(f_T + \mu)(\mu + \mu_T + r_T) - f_T r_T} \times \begin{bmatrix} (\mu + \mu_T + r_T) & r_T \\ f_T & (f_T + \mu) \end{bmatrix}. \tag{11}$$

By definition, $\mathcal{R}_T = \rho(\mathbf{K}) = \rho(-\mathbf{T}\mathbf{\Theta^{-1}})$, where $\rho(\mathbf{A})$ denotes the spectral radius of matrix $\mathbf{A}$. Therefore, the basic reproduction number of TB is

$$\mathcal{R}_T = \frac{\beta_T f_T}{(f_T + \mu)(\mu + \mu_T + r_T) - f_T r_T}. \tag{12}$$

4.1.2 Infection-free equilibrium and stability analysis

Sub-model (5) has an IFE Q_0^T obtained by setting its equations equal to zero

$$Q_0^T = (S_0, I_{T_0}, D_0) = (\frac{b}{\mu}, 0, 0). \tag{13}$$

Theorem 2 regarding this IFE is adapted from [20, 49].

Theorem 2. *The IFE Q_0^T is locally asymptotically stable when $R_T < 1$ and unstable when $R_T > 1$.*

Proof. We first find the Jacobian matrix of the linearised infected subsystem (7) evaluated at the IFE Q_0^T. If the trace of this Jacobian is negative and the determinant positive, its eigenvalues will have negative real parts, implying local asymptotic stability of Q_0^T.

First, we calculate the Jacobian matrix

$$J(Q_0^T) = \begin{bmatrix} \frac{\partial \dot{I}_T}{\partial I_T} & \frac{\partial \dot{I}_T}{\partial D} \\ \frac{\partial \dot{D}}{\partial I_T} & \frac{\partial \dot{D}}{\partial D} \end{bmatrix} \tag{14}$$

$$(Q_0^T) = \begin{bmatrix} -(f_T + \mu) & r_T + \beta_T \\ f_T & -(\mu + \mu_T + r_T) \end{bmatrix}$$

with trace $\text{tr}[J(Q_0^T)] = -(f_T + r_T + 3\mu + \mu_T) < 0$, and determinant

$$\det[J(Q_0^T)] = (f_T + \mu)(\mu + \mu_T + t_T) - f_T(r_T + \beta_T).$$

Then, $\det(J(Q_0^T)) > 0$ when $f_T(r_T + \beta_T) > (f_T + \mu)(\mu + \mu_T + r_T)$ i.e., when

$$\frac{\beta_T f_T}{(\mu + f_T)(\mu + \mu_T + r_T) - f_T r_T} = \mathcal{R}_T < 1.$$

This means that when $\mathcal{R}_T < 1$, both solutions of $\det[J(Q_0^T) - \lambda] = 0$ have negative real parts, and therefore Q_0^T is locally asymptotically stable when $\mathcal{R}_T < 1$ [20][6]. In contrast, when $\mathcal{R}_T > 1$, the determinant of $J(Q_0^T)$ is negative, implying that one eigenvalue is positive and the fixed-point Q_0^T is unstable. □

Thus, local stability of the IFE depends entirely on $\mathcal{R}_T$, which is in turn dependent on TB's transmission, progression and death parameters.

Further, when the IFE is locally stable, it is also globally stable if two conditions are met. We show this by

[6] See [20] for a formal proof that the IFE (referred to as a disease-free equilibrium in that paper) is locally asymptotically stable if all eigenvalues of the Jacobian have negative real parts.

following the method outlined in [50]. Rewrite the system of equations (5) as

$$\begin{aligned} \frac{dX}{dt} &= F(X,Z) \\ \frac{dZ}{dt} &= G(X,Z), \end{aligned} \tag{15}$$

where $X = (S) \in \mathbb{R}_+$ and contains states representing uninfected individuals, and $Z = (I_T, D) \in \mathbb{R}^2_+$ contains states representing infected individuals. Infection cannot 'suddenly' appear in a system with no infection or disease, thus $G(X,0) = 0$. This is true in subsystem (15), since $\lambda_T = \frac{\beta_T D}{N}$. Thus, the IFE is given by $Q_0^T = (X^*, 0)$, where $X^* = \left(\frac{b}{\mu}\right)$. Then, the IFE must be stable and conditions (H1) and (H2) must both be met to guarantee global asymptotic stability.

$$\begin{aligned} &\text{(H1) } X^* \text{ is globally asymptotically stable} \\ &\qquad \text{for } \frac{dX}{dt} = F(X,0) \\ &\text{(H2) } G(X,Z) = BZ - \hat{G}(X,Z),\ \hat{G}(X,Z) \geq 0 \\ &\qquad \text{for } (X,Z) \in \Omega_T \end{aligned} \tag{16}$$

where Ω_T is the biologically feasible region defined in equation (6), and $B = D_Z G(X^*, 0)$. Note that B is the same as the 2×2 bottom left component matrix of the full Jacobian evaluated at the IFE, i.e. $B = D_Z G(X^*, 0) = J(Q_0^T)$ in equation (14). Thus, B is a Metzler matrix with off-diagonal components non-negative. We have then already shown that both eigenvalues of B have negative real parts when $\mathcal{R}_T < 1$.

We can then find $\hat{G}(X,Z)$

$$\hat{G}(X,Z) = BZ - G(X,Z) = \begin{bmatrix} \beta_T D(1 - \frac{S}{N}) \\ 0 \end{bmatrix},$$

and is always non-negative since $\frac{S}{N} \leq 1$.

System (15) satisfies conditions (H1) and (H2), therefore the TB-only sub-model (5) also satisfies (H1) and (H2). The IFE of the TB-only sub-model is then globally asymptotically stable by Theorem 3.

Theorem 3. *If a system satisfies (H1) and (H2), has a locally asymptotically stable infection-free fixed point (i.e., its basic reproduction is less than unity), and B is a Metzler matrix whose eigenvalues have negative real parts, then this fixed point is* globally asymptotically stable*.*

Proof. We know that the infection-free point is locally asymptotically stable if the basic reproduction number is less than unity by Theorem 2. Let $Z_0 = Z(0)$. Thus, if $Z_0 > 0$, then $Z(t) \geq 0$. Further, e^{Bt} is a positive semigroup because B is a Metzler matrix[7]. Thus, using variation of parameters, we have

$$\begin{aligned} 0 \leq Z(t) &= e^{Bt} Z_0 - \int_0^t e^{B(t-s)} \hat{G}(X(s), Z(s)) ds \\ &\leq e^{Bt} Z_0. \end{aligned}$$

All eigenvalues of B have negative real parts when the basic reproduction number is less than unity. Therefore,

$$\lim_{t \to \infty} ||e^{Bt}|| = 0 \implies \lim_{t \to \infty} Z(t) = 0.$$

X^* is a globally asymptotically stable equilibrium of $\frac{dx}{dt} = F(X,0)$, which is itself a limiting system of $\frac{dx}{dt} = F(X(t), Z(t))$. Thus

$$\lim_{t \to \infty} X(t) = X^*$$

and $\lim_{t \to \infty}(X, Z_1, Z_2)$ is the IFE, thus the IFE is a globally asymptotically stable equilibrium point of the system. □

4.1.3 Endemic equilibrium and stability analysis

The endemic equilibrium of the TB sub-model is denoted $Q_*^T = (S^*, I_T^*, D^*)$, and is an equilibrium point in the system where TB is *endemic*—at least one of the infected compartments I_T^*, D^* is non-zero at the fixed point. At equilibrium, we have $\lambda_T^* g(\lambda_T^*) = 0$, where $\lambda_T^* = 0$ corresponds to the IFE and $g(\lambda_T^*) = 0$ corresponds to the existence of an endemic equilibrium. To find conditions for this equilibrium, sub-model equations (5) are solved in terms of the force of infection at steady state

$$\lambda_T^* = \frac{\beta_T D^*}{N^*}. \tag{17}$$

We then set model equations (5) to zero and find state values by substituting λ_T^* for λ_T

$$\begin{aligned} S^* &= \frac{b}{\mu + \lambda_T^*} \\ I_T^* &= \frac{\lambda_T^* b}{C}[(\mu + f_T)(\mu + \mu_T + r_T) + r_T(1 - f_T)] \\ D^* &= \frac{f_T \lambda_T^* b(\mu + f_T)}{C} \end{aligned} \tag{18}$$

where $C = (\mu + \lambda_T^*)(\mu + f_T)[(\mu + f_T)(\mu + \mu_T + r_T) - f_T r_T]$ is a common denominator for I_T^* and D^*. Thus, total population at equilibrium N^* is

$$\begin{aligned} N^* &= S^* + I_T^* + D^* \\ &= \frac{b}{C}[A(\mu + f_T + \lambda_T^*) + \lambda_T^*(r_T + f_T)] \end{aligned} \tag{19}$$

with $A = (\mu + f_T)(\mu + \mu_T + r_T) - f_T r_T$. Note that A is the denominator of the expression for $\mathcal{R}_T$, so $\mathcal{R}_T = \frac{\beta_T f_T}{A}$.

Then, we can find an expression for λ_T^* by substituting equations (18) and (19) into equation (17) and using

[7] A common property of Metzer matrices is that the exponential of a Metzler matrix is a non-negative matrix. For a full proof, see [51] (page 49).

the fact that at an *endemic* equilibrium, $\lambda_T^* \neq 0$

$$\lambda_T^* = \frac{(\mu + f_T)(\beta_T f_T - A)}{A + r_T + f_T}. \tag{20}$$

With some manipulation, we can then write λ_T^* in terms of $\mathcal{R}_T$

$$\lambda_T^* = B(\mathcal{R}_T - 1),$$
$$\text{with } B = \frac{(\mu+f_T)[\mu(\mu+\mu_T+r_T)+f_T(\mu+\mu_T)]}{\mu(\mu+\mu_T+r_T)+f_T(\mu+\mu_T+1)+r_T}. \tag{21}$$

Clearly, $B \geq 0$, thus the force of infection at equilibrium λ_T^* is positive only when $\mathcal{R}_T - 1 > 0$ i.e., when $\mathcal{R}_T > 1$. This has proved Theorem 4.

Theorem 4. *The TB sub-model system (5) has a unique endemic equilibrium when $R_T > 1$.*

4.2 HIV-only Sub-model

We then analyse the sub-model including only HIV/AIDS by setting states describing TB, COVID-19 and co-infection to zero: $I_T = D = I_C = I_{TC} = D_C = I_{TH} = I_{HC} = I_{THC} = I_{TA} = D_A = A_C = I_{TAC} = D_{AC} = 0$. Further, $\lambda_T = \lambda_C = 0$, giving system (22) governing the dynamics of HIV

$$\begin{cases} \dot{S} = b - [\mu + \lambda_H]S \\ \dot{I}_H = \lambda_H S - [(\mu + \mu_H) + f_H]I_H + r_H A \\ \dot{A} = f_H I_H - [(\mu + \mu_H) + r_H]A \end{cases} \tag{22}$$

with non-negative initial conditions S_0, I_{H_0}, A_0, and $\lambda_H(t)$ and $N(t)$ defined as

$$\lambda_H(t) = \frac{\beta_H}{N}[I_H + A]$$
$$N(t) = S + I_H + A.$$

The main difference between the TB and HIV sub-models is illustrated by their respective λ functions—the HIV sub-model contains *two* infectious states (I_H and A), while the TB sub-model has only one. Further, infected compartment I_H experiences an increased death rate due to HIV-infection, μ_H, as do individuals in state A, while in the TB sub-model only those with active TBD (in state D) experience an increased death rate μ_T due to disease.

The sub-model system (22) is mathematically and biologically well-posed, and can be studied exclusively in the biologically feasible region

$$\Omega_H = \left\{ (S, I_H, A) \in (R)_+^3 : N \leq \frac{b}{\mu} \right\}. \tag{23}$$

Again, following Theorem 1 for the full model (4) and taking epidemiology into account, Ω_H is positively invariant.

4.2.1 HIV's basic reproduction number

The basic reproduction number for HIV, denoted $\mathcal{R}_H$, is the number of secondary HIV infections produced by a single individual with infectious HIV throughout their infectious lifespan (including after progression to severe HIV/ AIDS, if relevant). Following the methods in the previous section, to compute $\mathcal{R}_H$ we must differentiate between new HIV infections and other movements between compartments. In the HIV sub-model, there are two *infected* states, I_H and A, which are both also *infectious* states, however I_H is the only *state at infection*. As before, we examine only the *infected subsystem* containing states I_H and A to investigate production of new infections at the IFE using the NGM method, ignoring the negative eigenvalue of the Jacobian resulting from the $\dot{S}$ equation (it is the only compartment an individual can enter immediately after infection).

We begin by linearising the *infected subsystem* about the IFE. At the IFE, $I_H = A = 0$ because there is no HIV infection, and $S = N$. In the linearisation, $\lambda_T = \frac{\beta_T}{S}(I_H + A)$ and the linearisation is closed. The linearised *infected* subsystem is

$$\begin{cases} \dot{I}_H = [\beta_H + r_H]A - [\mu + \mu_H + f_H - \beta_H]I_H \\ \dot{A} = f_H I_H - [\mu + \mu_H + r_H]A. \end{cases} \tag{24}$$

This system can be described by a Jacobian matrix, which can be decomposed as the sum of the two-dimensional *transmission* and *transition* matrices $\mathbf{T}$ and $\mathbf{\Theta}$[8]. We again use the epidemiological interpretation of $\mathbf{T}$ as containing elements representing *new* infections into *states at infection*. Therefore,

$$\mathbf{T} = \beta_H \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}. \tag{25}$$

Epidemiologically, we interpret $\mathbf{T}$ as before: non-zero rows correspond to *states at infection*—here, I_H—and non-zero columns correspond to *states of infectiousness*—both I_H and A. We find the transition matrix $\mathbf{\Theta}$ and its inverse

$$\mathbf{\Theta} = \begin{bmatrix} -(\mu + \mu_H + f_H) & r_H \\ f_H & -(\mu + \mu_H + r_H) \end{bmatrix} \tag{26}$$

$$\mathbf{\Theta^{-1}} = \frac{-1}{(\mu + \mu_H + f_H)(\mu + \mu_H + r_H) - f_H r_H} \times \begin{bmatrix} \mu + \mu_H + r_H & r_H \\ f_H & \mu + \mu_H + f_H \end{bmatrix}. \tag{27}$$

Then, with $\mathbf{K} = -\mathbf{T\Theta^{-1}}$, we can find $\mathcal{R}_H$ by calculating the spectral radius of $\mathbf{K}$: $\mathcal{R}_H = \rho(\mathbf{K})$. HIV's basic reproduction number in this model is

$$\mathcal{R}_H = \frac{\beta_H(\mu + \mu_H + r_H + f_H)}{(\mu + \mu_H + f_H)(\mu + \mu_H + r_H) - f_H r_H}. \tag{28}$$

[8] Here, the same notation is used as for the TB case for consistency, however matrices $\mathbf{T}$ and $\mathbf{\Theta}$ for HIV are different from those with the same names for TB.

4.2.2 Infection-free equilibrium and stability analysis

Model (22) has an IFE Q_0^H obtained by setting all equations to zero

$$Q_0^H = (S_0, I_{H_0}, A_0) = \left(\frac{b}{\mu}, 0, 0\right). \tag{29}$$

Theorem 5 is similar to Theorem 2 ensuring the existence of a locally asymptotically stable equilibrium when $\mathcal{R}_H < 1$.

Theorem 5. *The IFE Q_0^H is locally asymptotically stable when $\mathcal{R}_H < 1$ and unstable when $\mathcal{R}_H > 1$.*

The proof for this theorem is structurally the same as that for Theorem 2 and is therefore excluded from this paper.

Global stability of the IFE in this system is again guaranteed only when the IFE is locally stable and under the two conditions (H1) and (H2) stated in (16) in the previous section. To show that Theorem 3 applies to the HIV-only subsystem on region Ω_H, we rewrite the subsystem (22) [9]:

$$\begin{aligned} \frac{dX}{dt} &= F(X, Z) \\ \frac{dZ}{dt} &= G(X, Z), \end{aligned} \tag{30}$$

where $X = (S) \in \mathbb{R}_+$ contains uninfected states, while $Z = (I_H, A) \in \mathbb{R}_+^2$ contains infected states, and $G(X, 0) = 0$. Then, the IFE of this sub-model is $Q_0^H = (X^*, 0)$.

The Metzler matrix B for this system is

$$\begin{aligned} B &= D_Z G(X^*, 0) \\ &= \begin{bmatrix} \beta_H - \mu - \mu_H - f_H & \beta_H + r_H \\ f_H & -(\mu + \mu_H + r_H) \end{bmatrix}, \end{aligned}$$

with eigenvalues
$\gamma_\pm = \frac{1}{2}\left(-b \pm \sqrt{b^2 - 4\beta_H(\mu + \mu_H + r_H + f_H)(\frac{1}{\mathcal{R}_H} - 1)}\right)$,
where $b = 2\mu + 2\mu_H + f_H + r_H - \beta_H$. The real parts of B's eigenvalues are negative when $\mathcal{R}_H < 1$.

We then find

$$\hat{G}(X, Z) = BZ - G(X, Z) = \begin{bmatrix} \beta_H(I_H + A)(1 - \frac{S}{N}) \\ 0 \end{bmatrix},$$

which is non-negative since $\frac{S}{N} \leq 1$.

Therefore, system (30) satisfies conditions (H1) and (H2) from (16), as does the HIV-only subsystem (22). The IFE of the HIV-only sub-model is then globally asymptotically stable by Theorem 3.

[9] Again, functions are labelled in the same way as for the TB case, however, these functions F and G are *not* the same as those for TB.

4.2.3 Endemic equilibrium and stability analysis

The endemic equilibrium Q_*^H of the HIV sub-model exists if there is some equilibrium point in the system where at least one of the two infected compartments I_H, A is non-zero. State values at the endemic equilibrium can be found using the same methods as for the TB sub-model in the previous analysis, using the force of infection at equilibrium $\lambda_H^* = \frac{\beta_H}{N^*}(I_H^* + A^*)$

$$\begin{aligned} Q_*^H &= (S^*, I_H^*, A^*) \\ &= \left(\frac{b}{\mu + \lambda_H^*}, \frac{b\lambda_H^*(\mu + \mu_H + r_H)}{(\mu + \lambda_H^*)[P - f_H r_H]}, \frac{b\lambda_H^* f_H}{(\mu + \lambda_H^*)[P - f_H r_H]}\right) \end{aligned} \tag{31}$$

where $P = (\mu + \mu_H + r_H)(\mu + \mu_H + f_H)$. Note that I_H^* and A^* share a common denominator containing $P - f_H r_H$, which is the denominator in the expression for $\mathcal{R}_H$ in equation (28). We can thus write the equations for the states at equilibrium in terms of $\mathcal{R}_H = \frac{\beta_H}{P - f_H r_H}(\mu + \mu_H + r_H + f_H)$

$$\begin{aligned} &Q_*^H = \\ &\left(\frac{N}{\mathcal{R}_H}, \frac{(bR_H - \mu N)(\mu + \mu_H + r_H)}{\mathcal{R}_H(P - f_H r_H)}, \frac{f_H(b\mathcal{R}_H - \mu N)}{\mathcal{R}_H(P - f_H r_H)}\right). \end{aligned} \tag{32}$$

The following theorem provides conditions for existence of endemic equilibrium Q_*^H.

Theorem 6. *Endemic equilibrium Q_*^H exists when $R_H > 1$.*

Proof. HIV has an endemic equilibrium if $I_H, A > 0$ and $\dot{I}_H, \dot{A} = 0$. Applying these equalities to the HIV-only system equations for $\dot{I}_H$ and $\dot{A}$ from model (22) gives

$$\begin{aligned} 0 = \dot{A} &= f_H I_H - (\mu + \mu_H + r_H)A \\ &\implies A = \frac{f_H I_H}{\mu + \mu_H + r_H} \\ 0 = \dot{I}_H &= \frac{\beta_H S}{N}(I_H + A) - (\mu + \mu_H + f_H)I_H + r_H A \\ &\implies I_H = \frac{\frac{\beta_H S}{N} I_H + \left(\frac{\beta_H S}{N} + r_H\right) A}{\mu + \mu_H + f_H}. \end{aligned}$$

Substituting the first equation for A into the second for I_H and using that, at the endemic equilibrium, $\frac{S}{N} < 1$, we find

$$\begin{aligned} I_H &< \frac{\beta_H I_H}{\mu + \mu_H + f_H} + \frac{f_H(\beta_H + r_H)I_H}{(\mu + \mu_H + f_H)(\mu + \mu_H + r_H)} \\ 1 &< \frac{\beta_H(\mu + \mu_H + r_H + f_H) + f_H r_H}{P} \\ 1 &< \frac{\beta_H(\mu + \mu_H + r_H + f_H)}{P - f_H r_H}. \end{aligned}$$

The right-hand side of the final inequality is exactly our expression for $\mathcal{R}_H$, thus we find a unique endemic equilibrium in the HIV-only sub-model (22) for $\mathcal{R}_H > 1$. □

4.3 TB/HIV Sub-model

We move to an analysis of the sub-model describing dynamics of TB and HIV, excluding COVID-19. This represents the dynamics of these two infections before the COVID-19 pandemic. The TB/HIV sub-model is defined by setting all compartments describing COVID-19 infection or co-infection to zero: $I_C = I_{TC} = D_C = I_{HC} = I_{THC} = A_C = I_{ATC} = D_{AC} = 0$. Then, $\lambda_C(t) = 0$, and the model system is:

$$\begin{cases} \dot{S} &= b - [\mu + \lambda_T + \lambda_H]S \\ \dot{I}_T &= \lambda_T S - [\mu + f_T + \sigma_{TH}\lambda_H]I_T + \mathcal{R}_T D \\ \dot{D} &= f_T I_T - [(\mu + \mu_T) + \mathcal{R}_T]D \\ \dot{I}_H &= \lambda_H S - [(\mu + \mu_T) + \sigma_{HT}\lambda_T + f_H]I_H + \mathcal{R}_H A \\ \dot{I}_{TH} &= \sigma_{TH}\lambda_T I_T + \sigma_{HT}\lambda_H I_H - \\ &\quad [(\mu + \mu_{TH}) + f_T + f_H]I_{TH} + \mathcal{R}_H I_{TA} \\ \dot{A} &= f_H I_H - [(\mu + \mu_{TH}) + \sigma_{HT}\lambda_T + \mathcal{R}_H]A \\ \dot{I}_{TA} &= \sigma_{HT}\lambda_T A + f_T I_{TH} - [(\mu + \mu_{TH}) + f_T + \mathcal{R}_H] \\ &\quad I_{TA} + \mathcal{R}_T D_A \\ \dot{D}_A &= f_T I_{TA} + f_T I_{TH} - [(\mu + \mu_{TH}) + \mathcal{R}_T]D_A \\ &\quad + \sigma_{TH}\lambda_H D \end{cases} \tag{33}$$

with

$$\lambda_T(t) = \frac{\beta_T}{N}[D + \kappa_{T,H} D_A] \tag{34}$$

$$\lambda_H(t) = \frac{\beta_H}{N}[I_H + A + \kappa_{H,T}(I_{TH} + I_{TA} + D_A)] \tag{35}$$

$$N(t) = S + I_T + D + I_H + I_{TH} + A + I_{TA} + D_A \tag{36}$$

Model (33) has non-negative initial conditions $S_0,\ I_{T_0},\ D_0,\ I_{H_0},\ I_{TH_0},\ A_0,\ I_{TA_0},\ D_{A_0}$. Finally, the region within which model (33) is biologically feasible is

$$\Omega_{TH} = \left\{(S, I_T, D, I_H, I_{TH}, A, I_{TA}, D_A) \in \mathbb{R}^8_+ : N \leq \frac{b}{\mu}\right\}$$

which is positively invariant (the proof for this is omitted but follows the same structure as that for Theorem 1).

4.3.1 Basic reproduction number

The basic reproduction number for this sub-model is denoted $\mathcal{R}_{TH}$, and epidemiologically represents the number of secondary TB *or* HIV infections resulting from a single infectious TBD *or* HIV case. Again, this is mathematically defined as a spectral radius. We apply the same methods as for the single-infection sub-systems: identify the *infected subsystem*; linearise it about the IFE; decompose the Jacobian into transmission matrix $\mathbf{T}$ and transition matrix Θ; and compute eigenvalues of $\mathbf{K} = -\mathbf{T}\boldsymbol{\Theta}^{-1}$.

The *infected subsystem* contains states $I_T,\ D,\ I_H,\ I_{TH},\ A,\ I_{TA},\ D_A$. At the IFE, all eight of these states are zero, and $S = N$. Therefore, linearising sub-model (33) about the IFE, taking small quadratic disturbances to be negligible, gives

$$\begin{cases} \dot{I}_T &= (\beta_T + \mathcal{R}_T)D - (\mu + f_T)I_T + \beta_T\kappa_{T,H}D_A \\ \dot{D} &= f_T I_T - (\mu + \mu_T + \mathcal{R}_T)D \\ \dot{I}_H &= \beta_H - \mu - \mu_H - f_H)I_H + (\beta_H + \mathcal{R}_H)A \\ &\quad + \beta_H\kappa_{H,T}(I_{TH} + I_{TA} + D_A) \\ \dot{I}_{TH} &= -(\mu + \mu_{TH} + f_T + f_H)I_{TH}\mathcal{R}_H I_{TA} \\ \dot{A} &= f_H I_H - (\mu + \mu_H + \mathcal{R}_H)A \\ \dot{I}_{TA} &= f_H I_{TH} - (\mu + \mu_{TH} + f_T + \mathcal{R}_H)I_{TA} + \mathcal{R}_H D_A \\ \dot{D}_A &= f_T I_{TA} + f_T I_{TH} - (\mu + \mu_{TH} + \mathcal{R}_H)D_A \end{cases} \tag{37}$$

This can be used to compute $\mathcal{R}_{TH}$ by differentiating between *new* infections and other transitions. In this sub-model, every state except S is *infected*, all of which are also *infectious* apart from I_T. Following the NGM method, we calculate the 8×8 transmission matrix $\mathbf{T}$

$$\mathbf{T} = \begin{bmatrix} 0 & \beta_T & 0 & 0 & 0 & 0 & \beta_T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_H & \kappa_{H,T}\beta_H & \beta_H & \kappa_{H,T}\beta_H & \kappa_{H,T}\beta_H \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{38}$$

From $\mathbf{T}$ and the linear system (37) we infer that $\boldsymbol{\Theta}$ is

$$\boldsymbol{\Theta} = \begin{bmatrix} -p_1 & \mathcal{R}_T & 0 & 0 & 0 & 0 & 0 \\ f_T & -p_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -p_3 & 0 & \mathcal{R}_H & 0 & 0 \\ 0 & 0 & 0 & -p_4 & 0 & \mathcal{R}_H & 0 \\ 0 & 0 & f_H & 0 & -p_5 & 0 & 0 \\ 0 & 0 & 0 & f_H & 0 & -p_6 & \mathcal{R}_T \\ 0 & 0 & 0 & f_T & 0 & f_T & -p_7 \end{bmatrix} \tag{39}$$

where

$p_1 = \mu + f_T$; $p_2 = \mu + \mu_T + \mathcal{R}_T$;
$p_3 = \mu + \mu_H + f_H$; $p_4 = \mu + \mu_{TH} + f_T + f_H$;
$p_5 = \mu + \mu_H + \mathcal{R}_H$; $p_6 = \mu + \mu_{TH} + f_T + \mathcal{R}_H$;
$p_7 = \mu + \mu_{TH} + \mathcal{R}_T$

As before, $\mathcal{R}_{TH}$ is the spectral radius of $\mathbf{K} = -\mathbf{T}\boldsymbol{\Theta}^{-1}$.

Matrix $\mathbf{K}$ has three eigenvalues[10]

$$\begin{aligned}
\gamma_1 &= 0 \\
\gamma_2 &= \frac{\beta_T f_t}{p_1 p_2 - f_T \mathcal{R}_T} \\
&= \frac{\beta_T f_t}{(\mu + f_T)(\mu + \mu_T + \mathcal{R}_T) - f_T \mathcal{R}_T} \\
&= \mathcal{R}_T \\
\gamma_3 &= \frac{\beta_H (p_5 + f_H)}{p_3 p_5 - f_H \mathcal{R}_H} \\
&= \frac{\beta_H (\mu + \mu_H + \mathcal{R}_H + f_H)}{(\mu + \mu_H + f_H)(\mu + \mu_H + \mathcal{R}_H) - f_H \mathcal{R}_H} \\
&= \mathcal{R}_H
\end{aligned}$$

where $\gamma_2 = \mathcal{R}_T$ and $\gamma_3 = \mathcal{R}_H$ correspond to the reproduction numbers for the TB- and HIV-only sub-models, respectively. Therefore, $\mathcal{R}_{TH}$ is given by

$$\mathcal{R}_{TH} = \max\{\mathcal{R}_T, \mathcal{R}_H\}. \tag{40}$$

4.3.2 Infection-free equilibrium and stability

The IFE of this sub-model is The IFE of this sub-model is

$$\begin{aligned}
Q_0^{TH} &= (S_0, I_{T_0}, D_0, I_{H_0}, I_{TH_0}, A_0, I_{TA_0}, D_{A_0}) \\
&= \left(\frac{b}{\mu}, 0, 0, 0, 0, 0, 0, 0\right).
\end{aligned}$$

We theorise that the IFE is locally asymptotically stable for $\mathcal{R}_{TH} < 1$ because we know that $\mathcal{R}_{TH} = \max\{\mathcal{R}_T, \mathcal{R}_H\}$ and that both the TB-only and HIV-only sub-models had locally asymptotically stable IFEs for basic reproduction numbers less than unity. To explore the stability of this point we turn to numerical simulations. The TB/HIV sub-model (33) is applied to Soweto, an urban complex in Gauteng, South Africa, with a high prevalence of both infections. In this research, Soweto is used as a region for an example model implementation where the infections can interact and where modelling can have substantial effects on our understanding of and ability to control these epidemics. Parameter values for Soweto[11] are taken from the literature and government-collected population data (Table 3). The population is initialised using recent prevalence values[12] (Table 4), creating an initial value problem.

The simulation is initialised with transmission parameters β_T and β_H chosen so that $\mathcal{R}_T, \mathcal{R}_H < 1$. Soweto transmission rates for both infections are too high for an exploration of IFE stability, thus we select values for β_T and β_H from Denmark (Table 5), a country with very low TB and HIV transmission rates. Although these values will not correspond to realistic rates in the Soweto population, they allow for an examination of the IFE using transmission rates that are at least plausible in other contexts. Using these transmission rates with parameter values cited in Table 3 and equations (12) and (28), we calculate $\mathcal{R}_T = 0.002$ and $\mathcal{R}_H = 0.009$. Since $\mathcal{R}_T > \mathcal{R}_H$, $\mathcal{R}_{TH} = \mathcal{R}_T$ by equation (40).

Table 3: Parameter values for the TB/HIV sub-model (33) (modification parameter $\kappa_{H,T}$ could not be found in the literature so was set to 1 i.e., no effect).

Symbol	Value[13]	Year	Source
b	0.02	2016	[53]
μ	0.016	2019	[54]
β_T	0.26	2001	[55]
f_T	0.09	2001	[55]
$\mathcal{R}_T$	0.77	2015	[56]
μ_T	0.24	2014	[34]
$\kappa_{T,H}$	0.24	2000	[57]
σ_{TH}	1	2016	[58]
β_H	0.03	2004–2018	[59, 60]
f_H	0.22	2001	[61]
$\mathcal{R}_H$	2	2020	[62]
μ_H	0.3	2006	[63]
σ_{HT}	20	2012	[64]
μ_{TH}	1.6	2008	[32]

Model (33) is coded in Python with Runge-Kutta numerical integration to calculate the number of people per compartment over time. Simulation results are plotted in Figure 2. Simulation length is selected to allow dynamics to settle decidedly to equilibrium.

Trajectories illustrate the return of the system to the IFE after an introduction of infection into the population. Co-infected compartments I_{TH}, I_{TA} and D_A exhibit qualitatively similar behaviour: they decrease quickly to zero from the initial infected proportion and remain there. The other co-infected class I_{TH} shows similar behaviour, but the decrease occurs at a slower rate so the compartment takes longer to reach an infection-free state. Further, D and A exhibit an initial increase in infections before declining to zero. The increase in D is greater than that in A and the decay slower; the compartment takes the full 100-years to reach zero. In general, the rate of change of I_T appears similar to that of D—both exhibit slow exponential decay and only reach zero near the end of the simulation. However, the initial proportion of I_T is much greater and it experiences only a very small initial increase before decay begins. Finally, the susceptible class S displays an increase from its initial value to containing all members of the population.

[10] Steps for these calculations are excluded from this paper but follow the general method for eigenvalue calculations.

[11] Where possible, values are for Soweto itself, otherwise rates are found for Gauteng or South Africa and considered to apply to Soweto.

[12] Where necessary, reported incidence was converted to prevalence using the *prevalence = (incidence rate)* × *(mean infection duration)*, with infection durations from [52].

Table 4: State initial conditions for TB and HIV in the Soweto population.

Description	Model States	Value	Year	Source
Soweto total population	N_0	1271628	2011	[65]
Total national adult HIV prevalence	$I_{H_0}A_0 + I_{TA_0} + D_{A_0}$	19.5%	2021	[66]
Proportion of HIV cases classified as AIDS	$A_0 + I_{TA_0} + D_{A_0}$	12.7%	2013	[67]
Proportion of adults with TBD	$D_0 + D_{A_0}$	0.852%	2018	[68]
Proportion of LTBI HIV-negative adults	I_{T_0}	14.1%	2020	[69]
Proportion of LTBI HIV-positive adults	$I_{TH_0} + I_{TA_0}$	21.6%	2020	[69]
Proportion of adult AIDS cases with LTBI	I_{TA_0}	43.6%	2020	[70]
Proportion of AIDS cases with TBD	D_{A_0}	11.55%	2020	[70]

Table 5: Effective TB and HIV contact rates in Denmark.

Effective Annual Contact Rate	Value	Reference
β_T	8×10^{-5}	WHO EU monitoring
β_H	2.48×10^{-5}	[71]

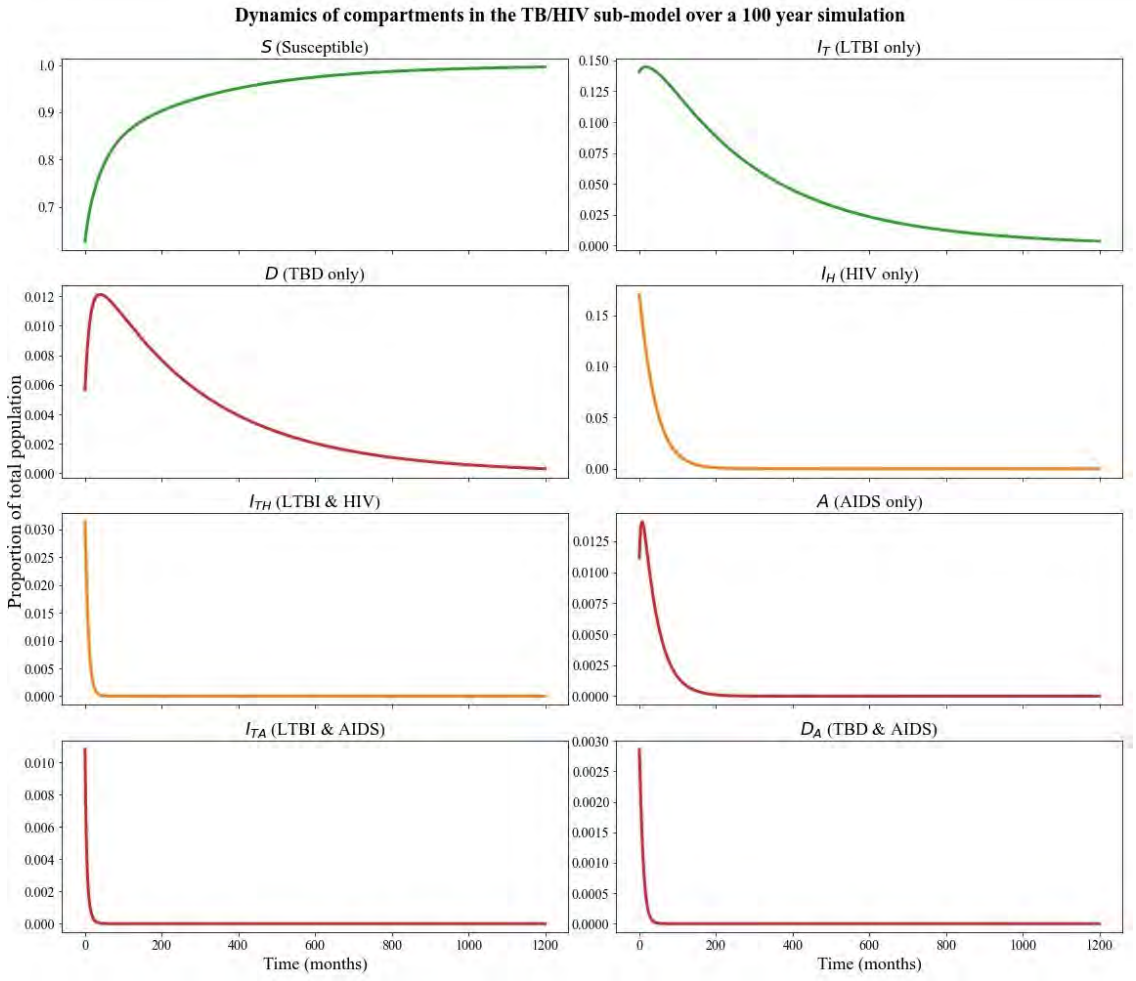


Figure 2: TB/HIV dynamics in the Soweto population after initialisation with current infection proportions and Denmark infection rates. Projections are coloured by infectiousness (non-infectious: green, HIV-only infectious: orange, TB- or severe HIV/AIDS-infectious: red).

4.3.3 Infection Invasion

We showed that systems with a single infection (TB or HIV) have globally asymptotically stable IFEs when the basic reproduction number is less than unity. However, in a model with two infections (i.e., both TB and HIV, as well as their co-infections), having one infection already in the system likely makes it easier for the other to appear. For example, say TB is already endemic in some population, meaning $\mathcal{R}_T > 1$, and the system is at the endemic equilibrium for TB. We know that HIV can invade the population if its basic reproduction number is greater than unity. However, this is not the same as the $\mathcal{R}_H$ calculated in Section 4.2 because we are starting from a TB-endemic equilibrium rather than an IFE. Thus, we must evaluate the Jacobian at the endemic equilibrium point. To calculate this, we substitute TB endemic equilibrium values from Equations (18) for states $S^*,\ I_T^*,\ D^*$ into the Jacobian of the TB/HIV sub-model with no HIV-infection. In the TB/HIV sub-model, the TB-only endemic equilibrium Q_*^T is

$$\begin{aligned} Q_*^T &= (S^*, I_T^*, D^*, I_H^*, I_{TH}^*, A^*, I_{TA}^*, D_A^*) \\ &= \left(\frac{b}{\mu + \lambda_T^*},\ \frac{\lambda_T^* b}{C}[(\mu + f_T)(\mu + \mu_T + \mathcal{R}_T) + \mathcal{R}_T(1 - f_T)],\ \frac{f_T \lambda_T^* b(\mu + f_T)}{C},\ 0, 0, 0, 0, 0 \right). \end{aligned} \tag{41}$$

At this equilibrium, total population is $N^* = S^* + I_T^* + D^*$. Linearising system (33) about this equilibrium and neglecting small quadratic terms gives

$$\begin{cases} \dot{S} = b - \mu S^* - \frac{\beta_T S^*}{N^*}(D^* + D + \kappa_{T,H} D_A) - \frac{\beta_H S^*}{N^*}[I_H + A + \kappa_{H,T}(I_{TH} + I_{TA} + D_A)] \\ \dot{I}_T = \frac{\beta_T D^*}{N^*} S + \frac{\beta_T S^*}{N^*}(D^* + D + \kappa_{T,H} D_A) - (\mu + f_T)(I_T^* + I_T) - \frac{\sigma_{TH}\beta_H I_T^*}{N^*}[I_H + A + \kappa_{H,T}(I_{TH} + I_{TA} + D_A)] \\ \dot{D} = f_T(I_T^* + I_T) - (\mu + \mu_T + \mathcal{R}_T)(D^* + D) + \frac{\sigma_{TH}\beta_H D^*}{N^*}[I_H + A + \kappa_{H,T}(I_{TH} + I_{TA} + D_A)] \\ \dot{I}_H = \frac{\sigma_{HT}\beta_H S^*}{N^*}[I_H + A + \kappa_{T,H}(I_{TH} + I_{TA} + D_A)] - (\mu + \mu_H + f_H) I_H + \mathcal{R}_H A \\ \dot{I}_{TH} = \frac{\sigma_{TH}\beta_H I_T^*}{N^*}[I_H + A + \kappa_{H,T}(I_{TH} + I_{TA} + D_A)] + \frac{\sigma_{HT}\beta_T D^*}{N^*} I_H - (\mu + \mu_{TH} + f_T + f_H) I_{TH} + \mathcal{R}_H I_{TA} \\ \dot{A} = f_H I_H - (\mu + \mu_H + \mathcal{R}_H) A + \frac{\sigma_{HT}\beta_T D^*}{N^*} A \\ \dot{I}_{TA} = \frac{\sigma_{HT}\beta_T D^*}{N^*} A - (\mu + \mu_{TH} + f_T + \mathcal{R}_H) I_{TA} + \mathcal{R}_T D_A \\ \dot{D}_A = f_T I_{TA} + f_T I_{TH} - (\mu + \mu_{TH} + \mathcal{R}_T) D_A + \frac{\sigma_{TH}\beta_H D^*}{N^*}[I_H + A + \kappa_{H,T}(I_{TH} + I_{TA} + D_A)]. \end{cases} \tag{42}$$

Then, we can the Jacobian for the linearised system (42) at the endemic equilibrium

$$J(Q_*^T) = \begin{bmatrix} 0 & 0 & -q_1 S^* & -q_2 S^* & -\kappa_{H,T} q_2 S^* & -q_2 S^* & -\kappa_{H,T} q_2 S^* & -(\kappa_{T,H} q_1 + \kappa_{H,T} q_2) S^* \\ q_1 D^* & -p_1 & q_1 S^* & -\sigma_{TH} q_2 I_T^* & -q_3 I_T^* & -\sigma_{TH} q_2 I_T^* & -q_3 I_T^* & \kappa_{T,H} q_1 S^* - q_3 I_T^* \\ 0 & f_T & -p_2 & q_3 D^* & \sigma_{TH} q_2 D^* & \sigma_{TH} q_2 D^* & q_3 D^* & q_3 D^* \\ 0 & 0 & 0 & \sigma_{HT} q_2 S^* - p_3 & q_4 S^* & \sigma_{HT} q_2 S^* + \mathcal{R}_H & q_4 S^* & q_4 S^* \\ 0 & 0 & 0 & \sigma_{HT} q_1 D^* + \sigma_{TH} q_2 I_T^* & q_3 I_T^* - p_4 & \sigma_{TH} q_2 I_T^* & q_3 I_T^* + \mathcal{R}_H & q_3 I_T^* \\ 0 & 0 & 0 & f_H & 0 & \sigma_{HT} q_1 D^* - p_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{HT} q_1 D^* & -p_6 & \mathcal{R}_T \\ 0 & 0 & 0 & \sigma_{TH} q_2 D^* & q_3 D^* + f_T & \sigma_{TH} q_2 D^* & q_3 D^* + f_T & q_3 D^* - p_7 \end{bmatrix} \tag{43}$$

with

$$q_1 = \frac{\beta_T}{N^*}; \qquad q_2 = \frac{\beta_H}{N^*}; \qquad q_3 = \kappa_{H,T} \sigma TH q_2; \qquad q_4 = \kappa_{T,H} \sigma HT q_2.$$

The lower right 5×5 component of the Jacobian $J(Q_*^T)$ describing HIV-infected compartments decouples entirely. Using the same arguments as presented in section 4.1, we can examine only this 5×5 component to find transmission and transition matrices $\mathbf{T}$ and $\mathbf{\Theta}$

$$\mathbf{T} = \begin{bmatrix} \sigma_{HT} q_2 S^* & q_4 S^* & \sigma_{HT} q_2 S^* & q_4 S^* & q_4 S^* \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{44}$$

$$\mathbf{\Theta} = \begin{bmatrix} -p_3 & 0 & \mathcal{R}_H & 0 & 0 \\ \sigma_{HT} q_1 D^* + \sigma_{TH} q_2 I_T^* & q_3 I_T^* - p_4 & \sigma_{TH} q_2 I_T^* & q_3 I_T^* + \mathcal{R}_H & q_3 I_T^* \\ f_H & 0 & \sigma_{HT} q_1 D^* - p_5 & 0 & 0 \\ 0 & 0 & \sigma_{HT} q_1 D^* & -p_6 & \mathcal{R}_T \\ \sigma_{TH} q_2 D^* & q_3 D^* + f_T & \sigma_{TH} q_2 D^* & q_3 D^* + f_T & q_3 D^* - p_7 \end{bmatrix} \tag{45}$$

There are no zero columns in $\mathbf{T}$, which is expected as all states are infectious. The only non-zero row corresponds to I_H, the single state at infection. Because of this, we can already see that $\mathbf{K}$ will have one non-zero row, corresponding with one non-zero eigenvalue. This gives HIV's basic reproduction number for its introduction into a population that is entirely susceptible to HIV but already TB-infected. The eigenvalue of $\mathbf{K}$ was calculated, but the expression is particularly complex and could not easily be written in terms of $\mathcal{R}_H$, which would allow us to make definitive statements about HIV's spread in an infection-free population versus in a TB-infected population. Further manipulation of the expression is required to test the hypothesis that this basic reproduction number is greater than $\mathcal{R}_H$, and therefore that it becomes easier for a second infection to invade a population if one is already endemic.

5 Full Model Analysis

In this section, the full model is examined to find the basic reproduction number when all three infections are present in a population, as in the many high TB/HIV burden countries during the current COVID-19 pandemic. Numeric simulations of the model are performed for Soweto.

5.1 Basic Reproduction Number

Following a similar analysis as for the TB/HIV sub-model, we can find the basic reproduction number for the full model $\mathcal{R}_0$, mathematically defined as spectral radius $\rho(\mathbf{K})$. Analogously to the TB/HIV sub-model, the dominant eigenvalue of $\mathbf{K}$ is the maximum of the basic reproduction numbers for the individual infections—$\mathcal{R}_T$, $\mathcal{R}_H$, and $\mathcal{R}_C$ (with $\mathcal{R}_C$ the basic reproduction number for COVID-19 in a subsystem modelling only COVID-19 dynamics). Since the COVID-19 sub-model has a single infected compartment I_C, its basic reproduction number $\mathcal{R}_C$ is the product of its effective contact rate and mean duration of infection [20]. Mean duration of infection is the inverse of recovery rate $\mathcal{R}_C$, thus $\mathcal{R}_C = \frac{\beta_C}{\mathcal{R}_C}$. Therefore

$$\begin{aligned}\mathcal{R}_0 &= \max\{\mathcal{R}_T, \mathcal{R}_H, \mathcal{R}_C\} \\ &= \max\left\{\frac{\beta_T f_T}{(\mu+f_T)(\mu+\mu_T+r_T)-f_T r_T}, \right. \\ &\qquad \frac{\beta_H(\mu+\mu_H+r_H+f_H)}{(\mu+\mu_H+f_H)(\mu+\mu_H+r_H)-f_H r_H}, \\ &\qquad \left.\frac{\beta_C}{r_C}\right\}. \end{aligned} \tag{46}$$

5.2 Numeric Simulations

Numeric simulations of system (4) are conducted with TB and HIV parameter values and initial conditions for Soweto as cited in Tables 2 and 3, also applying additional COVID-19 parameter values listed in Tables 6 and 7[14]. Initialising this system with different basic reproduction numbers for the three infections allows for an investigation of system dynamics under different conditions.

Table 6: Definitions of parameters used in model (4).

Symbol	Value	Year	Source
β_C	0.57	2021	[72]
μ_C	$9.24 * 10^{-}5$	2021	[73]
$\mathcal{R}_C$	$1-\mu_C$		
μ_{TC}	0.21	2021	[74]
μ_{HC}	2.39	2020	[75]
μ_{THC}	2.7	2020	[75]
σ_{TC}	1.71	2020	[76]
σ_{HC}	1	2020	[77]

Table 7: COVID-19 states' initial conditions for the Soweto population (all values are proportions of the adult population from 2021 [78]).

Description	Model States	Value
HIV/COVID-19 co-infection	$I_{HC_0}+A_{C_0}+I_{TAC_0}$ $+I_{THC_0}+D_{AC_0}$	5.9%
TBD/COVID-19 co-infection	$D_{C0}+D_{AC_0}$	3.6%
TBD/AIDS/COVID-19 co-infection	D_{AC_0}	2.61%

5.2.1 Infection-free equilibrium ($\mathcal{R}_{THC}<1$)

First, behaviour of the system close to the IFE is explored for $\mathcal{R}_{THC}<1$, that is $\mathcal{R}_T, \mathcal{R}_H, \mathcal{R}_C < 1$ by equation (46). As before, we use transmission rates for TB and HIV from Denmark (Table 5) to fulfil $\mathcal{R}_T, \mathcal{R}_H < 1$. Initial conditions are described by Soweto prevalence values (Tables 4 and 8). Only COVID-19 co-infection classes whose population proportion was measured close to the beginning of the pandemic in South Africa (March 2020) are included; other COVID-19 classes are assumed to be zero at initialisation ($I_{TC_0} = I_{TAC_0} = A_{C_0} = I_{THC_0} = 0$). Figure 3 depicts dynamics of system (4) under these conditions, showing changes in proportions of the total population in each compartment over time. Non-infectious classes are plotted in green, TBD or AIDS-infected classes in red, HIV-infected classes in orange, and COVID-19 only infectious classes in blue.

[14] Modification parameters not listed here were not found in the literature and their values set to 1 in simulations.

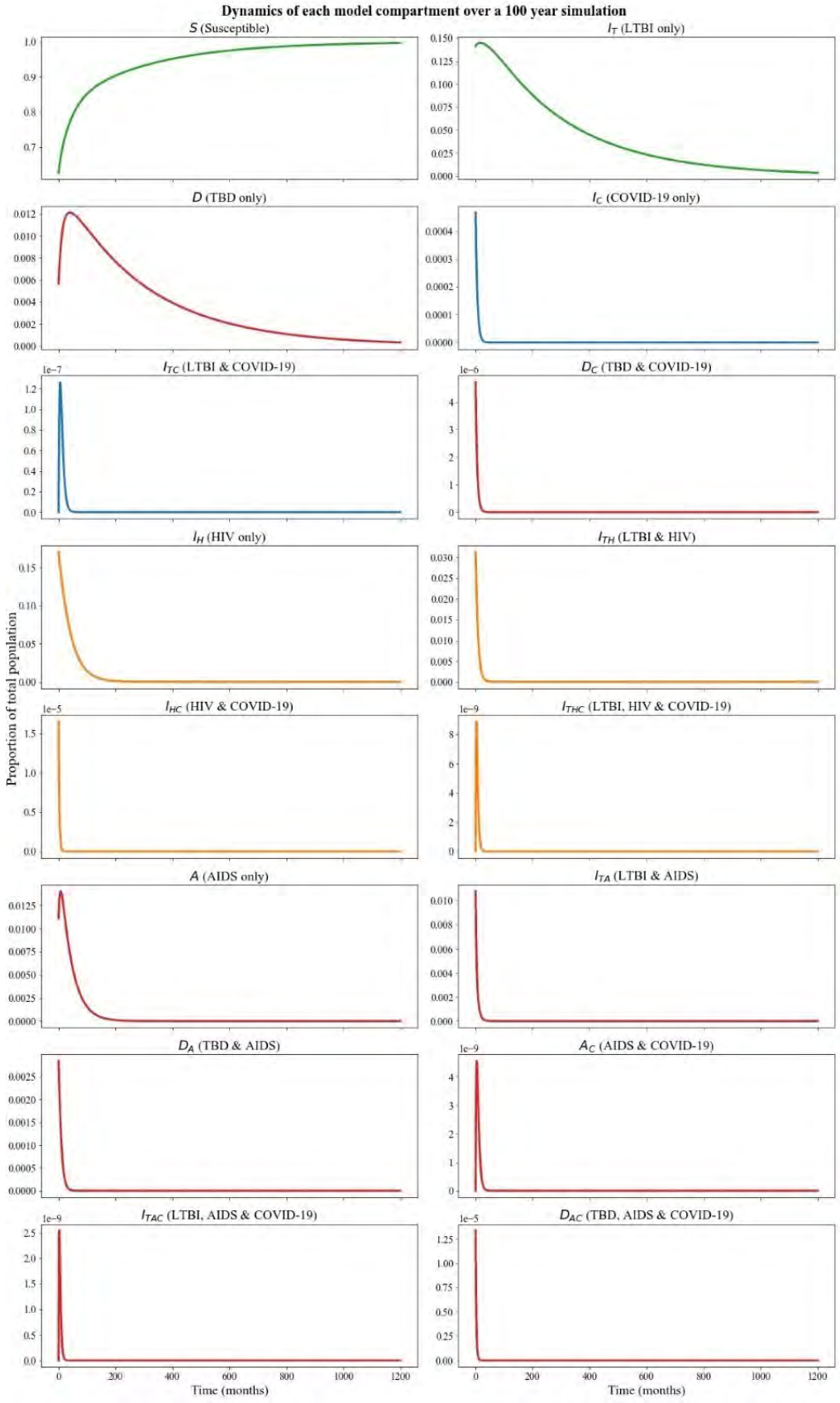


Figure 3: System dynamics of a simulation of model (4) with $\mathcal{R}_{THC} < 1$ and compartments coloured by infectious characteristics (non-infectious: green, COVID-19-only infectious: blue, HIV-only infectious: orange, TB- or severe HIV/AIDS-infectious: red).

Every infected compartment approaches zero after the initial disturbance, either after a period of growth (I_T, D, I_{TC}, I_{THC}, A, A_C, I_{TAC}) or immediately (I_C, D_C, I_H, I_{TH}, I_{HC}, I_{TA}, D_A, D_{AC}). The susceptible class tends to 1, implying the total population will be uninfected after 100 years with these transmission rates, and the system finds an IFE. This suggests that the IFE of model (4) is locally stable and attracting, but this is not conclusive and further exploration is required.

5.2.2 Realistic transmission rates ($\mathcal{R}_{THC} > 1$)

To examine the effects of introducing COVID-19 into Soweto, parameter and population prevalence values are used as above to initialise the model. For COVID-19, β_C is 50 per 100 000, which is classified by Centers for Disease Control and Prevention (CDC) as the boundary between low and moderate transmission for this pandemic [79], giving $\mathcal{R}_{THC} = \mathcal{R}_C = 3.44$. Figure 4 displays progressions of model compartments over the simulation.

We observe that all TB and COVID-19 infected classes find endemic equilibria after some time, while all HIV-infected and co-infected classes tend towards zero. Many COVID-19 co-infected classes exhibit a sharp increase in proportion soon after initialisation (I_{TC}, I_{HC}, I_{THC}, A_C, I_{TAC}, D_{AC}). This corresponds with a reduction in non-COVID classes I_T, I_H, I_{TH}, A, I_{TA} and D_A. Thus, it appears that individuals move from the non-COVID class to the corresponding COVID-infected one (for example, from I_T to I_{TC}, or from I_H to I_{HC}) as coronavirus spreads. The susceptible class S decreases as COVID-19 spreads rapidly, confirming that not all new cases are from those already infected with TB or HIV, but that the total population is affected by COVID-19. To observe trends for the three infections in more detail, we examine progressions of states involving each infection separately and plot their net effects. First, we examine TB: Figure 5a depicts developments of all LTBI and TBD states over time, and Figure 5b shows the net progression of TB in the population.

Figure 5a demonstrates an increase in TBD-only cases corresponding to a decrease in LTBI-only cases, implying the new TBD cases result from conversion of LTBI. The decrease in LTBI-only cases is larger than the increase in TBD, therefore it is likely that the rest of the decrease can be attributed to COVID-19-infection as it corresponds to a steep increase in 'LTBI and COVID-19'. In Figure 5a, we observe the stabilisation of all TB states; compartments that do not describe HIV stabilise at non-zero proportions, while those describing TB/HIV co-infection stabilise at zero. Net effects of TB are plotted in Figure 5b, demonstrating overall stabilisation of total LTBI and TBD in the population. After some initial fluctuation, both increase to a peak proportion and then decay towards an equilibrium (total LTBI at approximately 3.3% of the population, and total TBD at around 1.2%). Stabilisation appears to occur at around $t = 600$ months $= 50$ years.

We then analyse HIV using dynamics depicted in Figure 6. HIV states are plotted only over the first five years as this period contains all relevant dynamics; afterwards, HIV is no longer present in the population. In Figure 6a, HIV-only and AIDS-only classes display an initial period of little change for around 10 months. They both then decline, seemingly as the 'HIV and COVID-19' and the 'AIDS and COVID-19' classes increase, which corresponds to COVID-19 infection. After this, all classes reach zero by $t \approx 55$ months. Figure 6b confirms this, showing AIDS eradication at $t \approx 38$ months and HIV eradication at $t \approx 52$ months.

Dynamics of COVID-19 are illustrated in Figure 7. Figure 7a shows HIV/COVID-19 co-infected classes tending to zero, while COVID-only and TB/COVID-19 classes stabilise at nonzero proportions. COVID-19 only displays a high rate of initial increase before its growth slows at a population proportion of approximately 0.6 and the compartment decays before tending towards a proportion around 0.38. All classes stabilise just after $t = 600$ months. Figure 7b shows the net effect of COVID-19: here, at $t \approx 600$ months, the total number of COVID-19 cases stabilises at 0.75.

Under this model, we have observed the apparent eradication of HIV from the Soweto population with realistic parameter values, which is unlikely as HIV is incurable. Basic reproduction number $\mathcal{R}_H > 1$ indicates that a more plausible outcome would have been the stabilisation of proportions of HIV infections at some endemic equilibrium. There are two possible explanations for this elimination. The first is that introducing COVID-19 into the system causes increased excess deaths due to co-infection. We examine this by plotting cumulative excess deaths and excess death rate of modelled non-natural deaths (Figure 8). Here, we observe three peaks in excess death rate; the first two are in quick succession and particularly sharp, and the third is later and much smoother at a lower population value. Finally, excess death rate stabilises at around 2500, matching the linear increase in cumulative deaths. The first peak in death rate corresponds with the initial peak of TBD-only shown in Figure 4, the second with peaks in most COVID-19 classes, and the third with peaks in D and D_C. This implies that initially most infectious deaths are attributable to TBD. Eventually, as COVID-19 invades the population, deaths become attributed to co-infection of either TB or HIV with COVID-19. It is possible that these excess deaths of HIV co-infected patients result in the relatively quick eradication of HIV from the system. TBD, however, remains a significant cause of death, explaining the third, lower peak. By the end of the simulation, all excess deaths are attributed to either TBD or COVID-19.

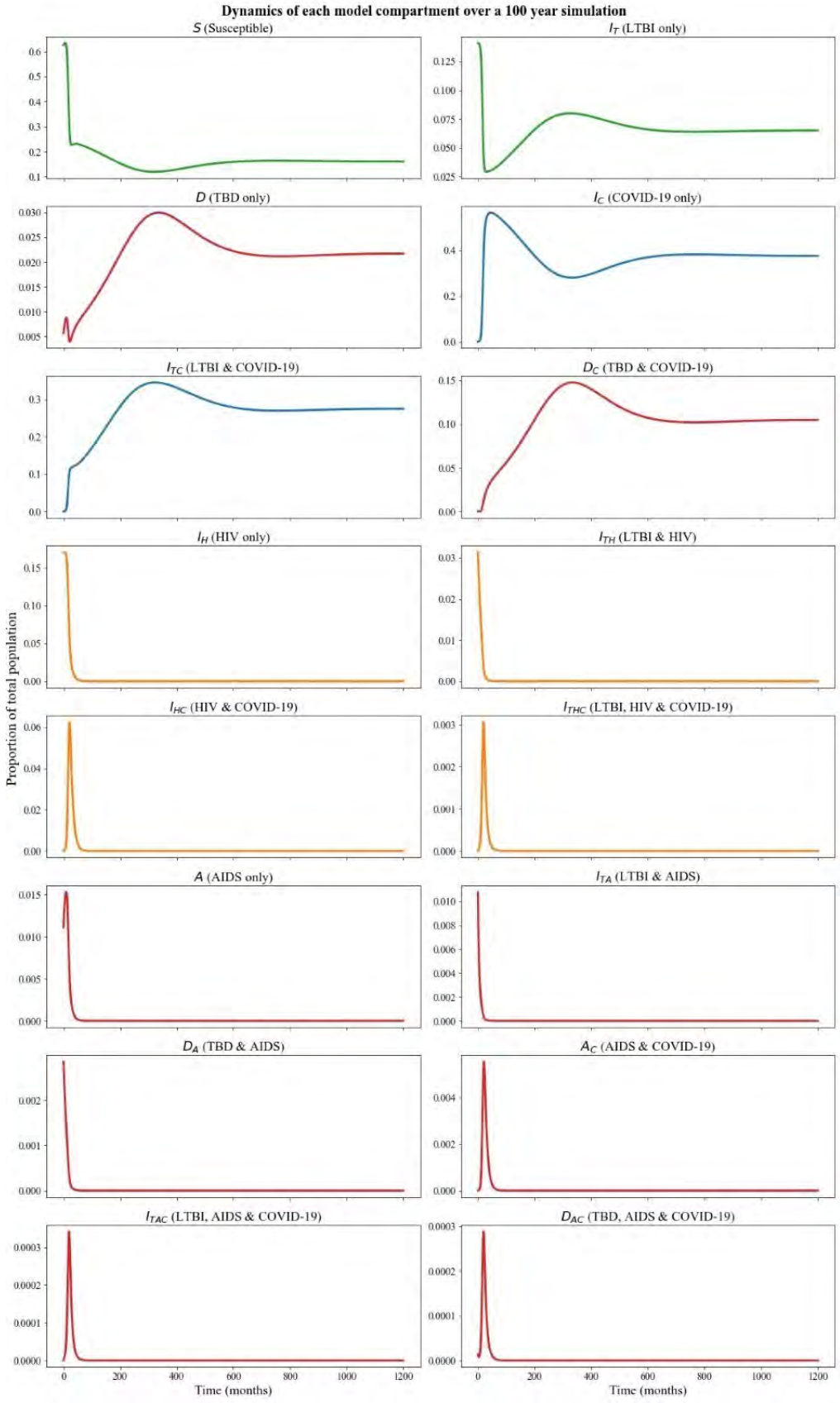
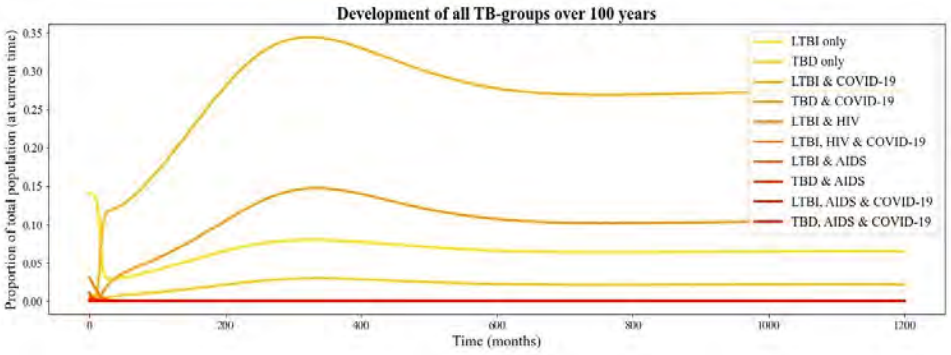


Figure 4: System dynamics following the introduction of COVID-19 in a population with TB/HIV already present. Compartments are coloured by infectious characteristics (non-infectious: green, COVID-19-only infectious: blue, HIV-only infectious: orange, TB- or severe HIV/AIDS-infectious: red).

(a) All TB classes



(b) LTBI and TBD totals

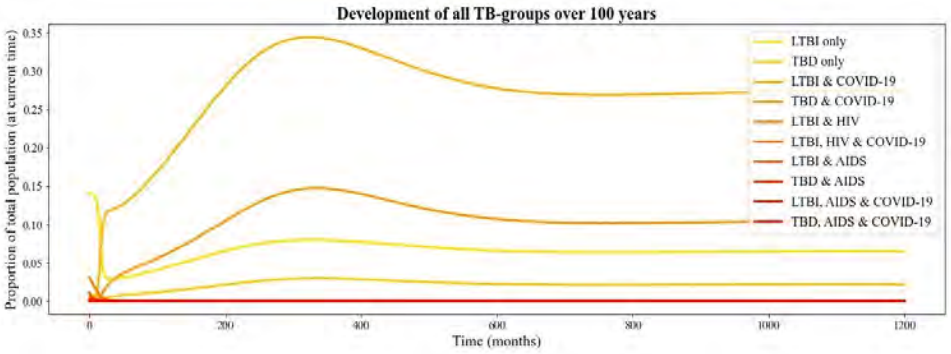
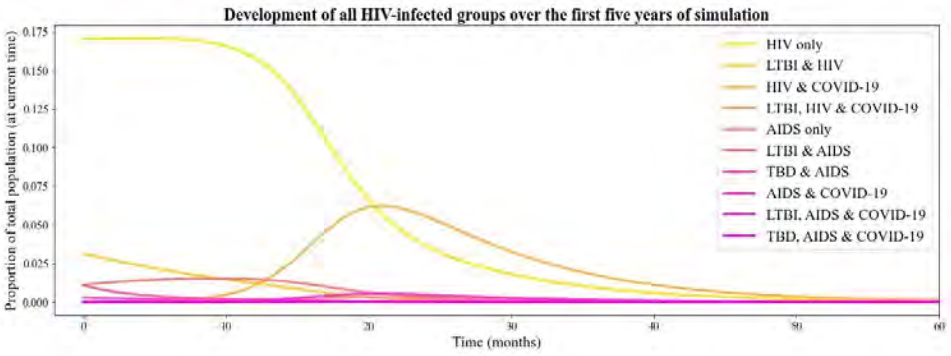


Figure 5: Dynamics of all TB classes (5a) and total TB (5b) plotted as population proportions over time.

(a) All HIV classes



(b) HIV (both HIV and AIDS) and AIDS totals

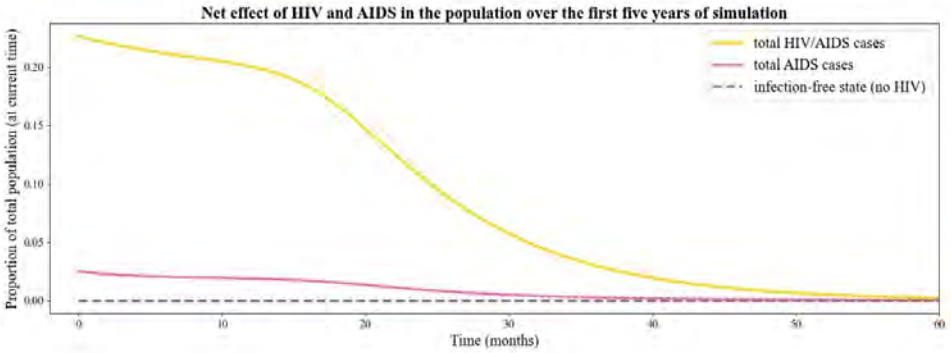
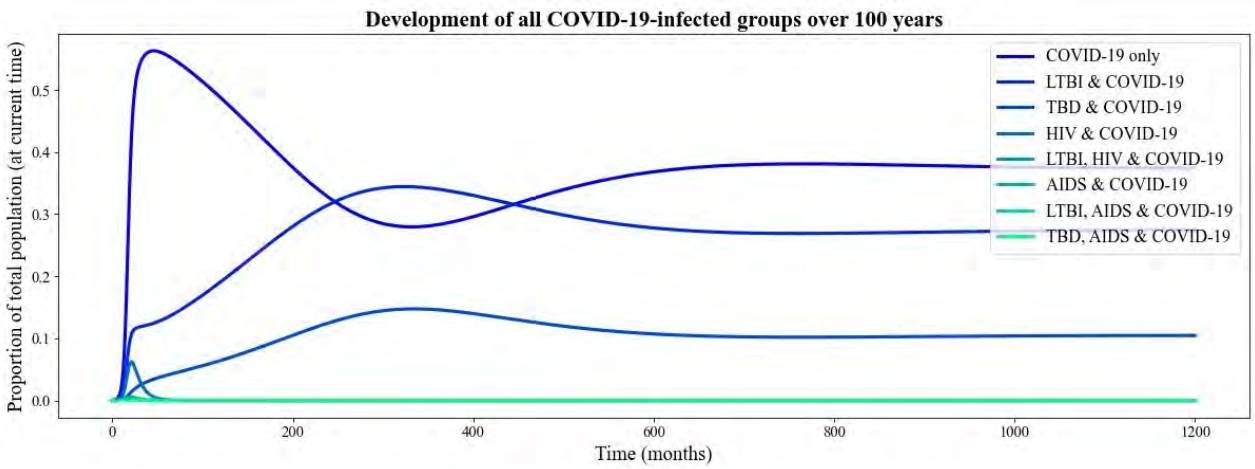


Figure 6: Dynamics of all HIV classes (6a) and total HIV (6b) plotted as population proportions over the first five years of the simulation.

(a) All COVID-19 classes



(b) Total COVID-19

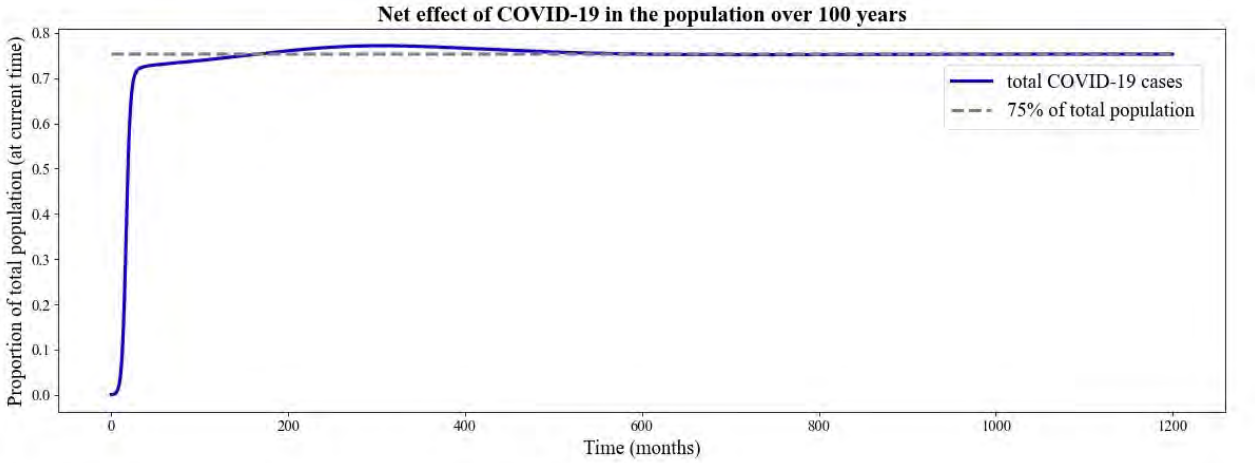


Figure 7: Dynamics of all COVID-19 classes (7a) and total COVID-19 (7b) in the population over time.

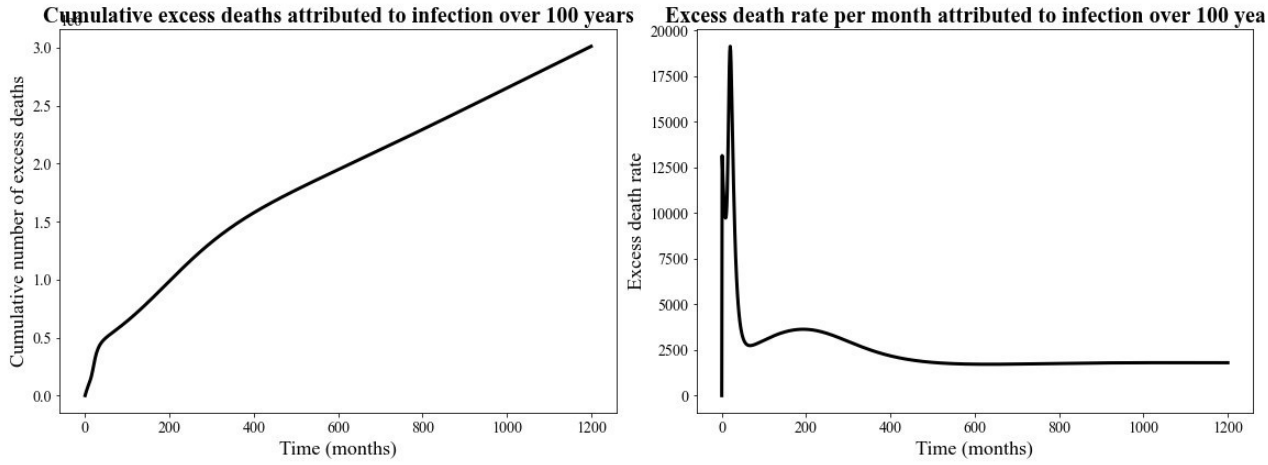


Figure 8: Cumulative excess deaths (left) and excess death rate (right) attributed to infection over the simulated period.

The second explanation is related to a model simplification. Model (4) does not include immunity from COVID-19. Further, to capture differences between acute COVID-19 and chronic TB and HIV, certain transmission paths are ignored in this model. This combination means it is possible for individuals to be continually re-infected with COVID-19 from any initial state. In cases where infection or disease progression were excluded while an individual is SARS-CoV-2-infected, this consistent re-infection essentially translates as protection from HIV, which is eventually removed from the population as infected individuals die without transmitting.

When COVID-19 is introduced into the Soweto population with existing endemic TB and HIV, we observe a stabilisation of COVID-19 infections, LTBI and TBD, alongside a complete eradication of HIV. Although this removal is unlikely to be representative of actual system dynamics, the eradication of HIV and the stabilisation of all remaining infections allows us to reduce the system to susceptibles S, total COVID-19 cases, total LTBI cases, and total TBD cases, all of which stabilise at a similar time in the simulation. Specifically, we examine the introduction and stabilisation of total COVID-19 in the phase space.

5.2.3 Phase Portraits

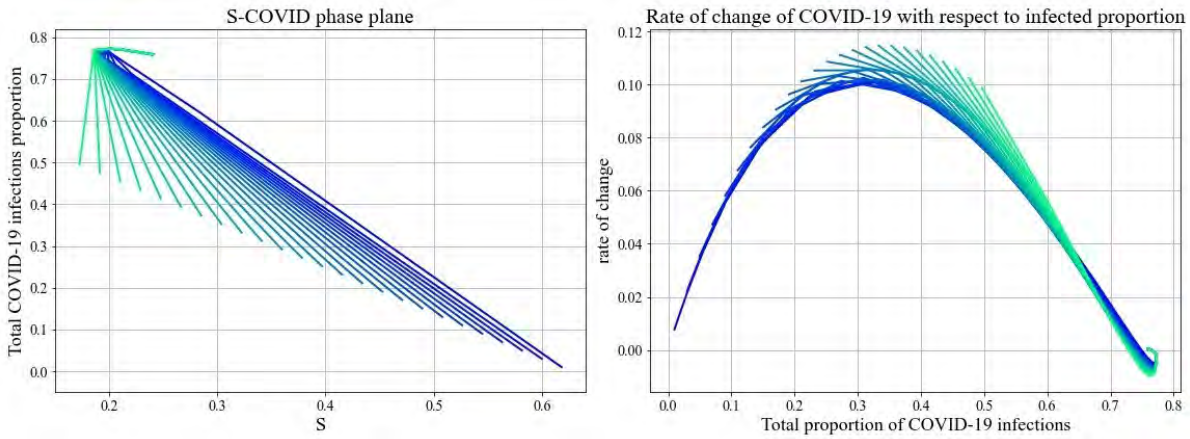
We explore differences in total COVID-19 equilibrium points found under different initial conditions of the system (4). We initialise the system with initial proportions of COVID-19 ranging from 1% to 50% of the total population. Resulting trajectories are plotted in the phase space of total COVID-19 and S, and the rate of change of COVID-19 plotted against its actual proportion. We found previously that the total number of COVID-19 cases stabilised at an endemic equilibrium (Figure 7b); these phase space plots then allow us to identify fixed points in the system using qualitative behaviour of trajectories with varying initial conditions. Plotting trajectories in the phase space for different values of the control parameter $\mathcal{R}_C$ allows us to examine how the position of the fixed point in the space depends on the system's basic reproduction number, using $\mathcal{R}_{THC} = \max\{\mathcal{R}_T, \mathcal{R}_H, \mathcal{R}_C\} = \mathcal{R}_C$ here. Phase plots are created for $\mathcal{R}_C$ decreasing from $\mathcal{R}_C = 4.4$ to $\mathcal{R}_C = 0.4$. Figure 9 shows these dynamics, with darker blue shades indicating lower initial COVID-19 proportions.

In Figure 9a we observe behaviour of COVID-19 qualitatively similar to that seen in the time domain example: all nearby trajectories tend towards a stable fixed point in the system at approximately 0.75, with $S \approx 0.24$. At this COVID-19 proportion there are no new cases, confirming this to be a fixed point. As $\mathcal{R}_C$ decreases, the fixed point begins to settle at lower proportions of COVID-19. For $\mathcal{R}_C > 1$ (Figures 9a–9d), this fixed point is an endemic equilibrium; infection dynamics stabilise at some non-zero proportion. In each case, we confirm that the endemic equilibrium is a fixed point as the derivative is zero at the COVID-19 proportion corresponding with the equilibrium point. Together, the rate of change trajectories shown on the right in Figures 9a–9d follow a parabolic shape, increasing to a maximum before decreasing and doubling back to zero at the fixed point.[15] Furthermore, the fixed point appears to be stable, as all nearby trajectories move towards it. Finally, in Figure 9e with $\mathcal{R}_C = 0.4 < 1$, the fixed point is at zero COVID-19 infections, indicating that COVID-19 is removed from the population at equilibrium. The rate of change of COVID-19 is always negative. At the COVID-free point in Figure 9e, the proportion of susceptibles S in the population does not occur at $S = 1$, implying that there is still some infection, but not COVID-19. This is due to TB in the system; $\mathcal{R}_T$ has not decreased with $\mathcal{R}_C$, thus TB is still endemic and the remaining proportion has LTBI or TBD.

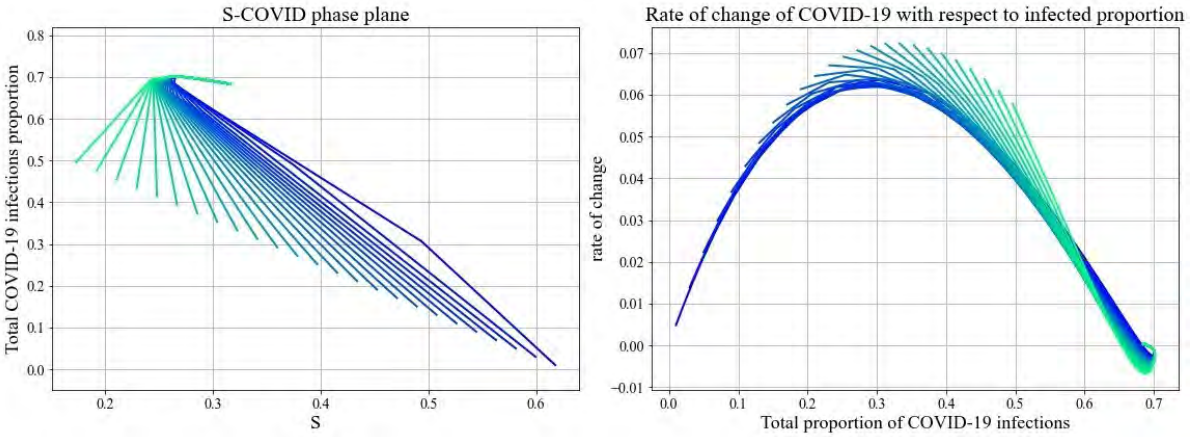
Plots in Figure 9 suggest that COVID-19 is removed when $\mathcal{R}_C < 1$ (Figure 9e), but when $\mathcal{R}_C > 1$, total COVID-19 settles at an endemic equilibrium. We confirm this using the bifurcation diagram in Figure 10, which illustrates the dependence of the COVID-19 fixed point on $\mathcal{R}_C$. Here, we observe that the fixed point is zero and COVID-19 is eradicated for $\mathcal{R}_C < 1$, but, as soon as $\mathcal{R}_C$ is increased past this critical point, the COVID-free equilibrium loses its stability and the system instead finds an endemic equilibrium. This indicates that $\mathcal{R}_C$ is a control parameter—if it can be decreased to below unity, COVID-19 will be removed from system 4.

[15] This behaviour is more obviously visible in Figures 9a–9c, and less so in 9d, but a similar behaviour is exhibited.

(a) Phase plots for $\mathcal{R}_C = 4.4$



(b) Phase plots for $\mathcal{R}_C = 3.4$



(c) Phase plots for $\mathcal{R}_C = 2.4$

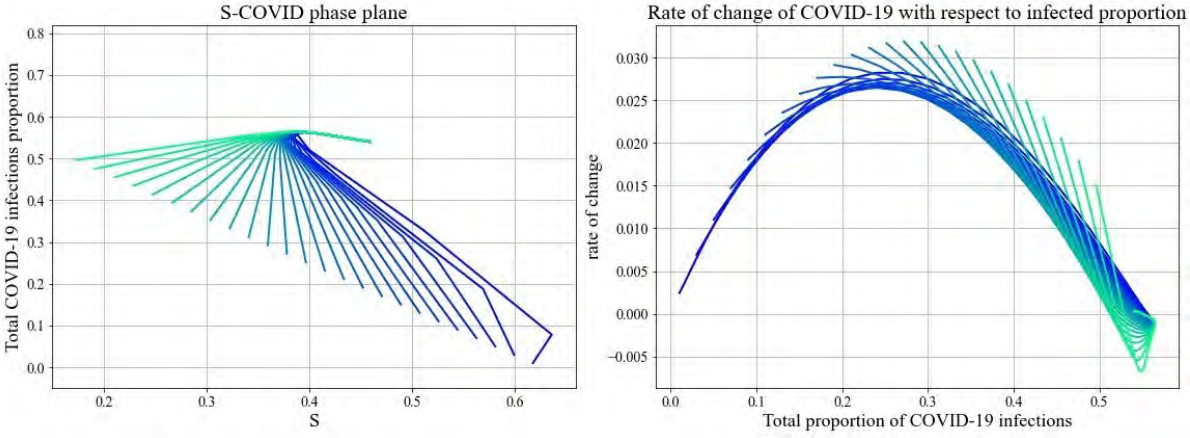
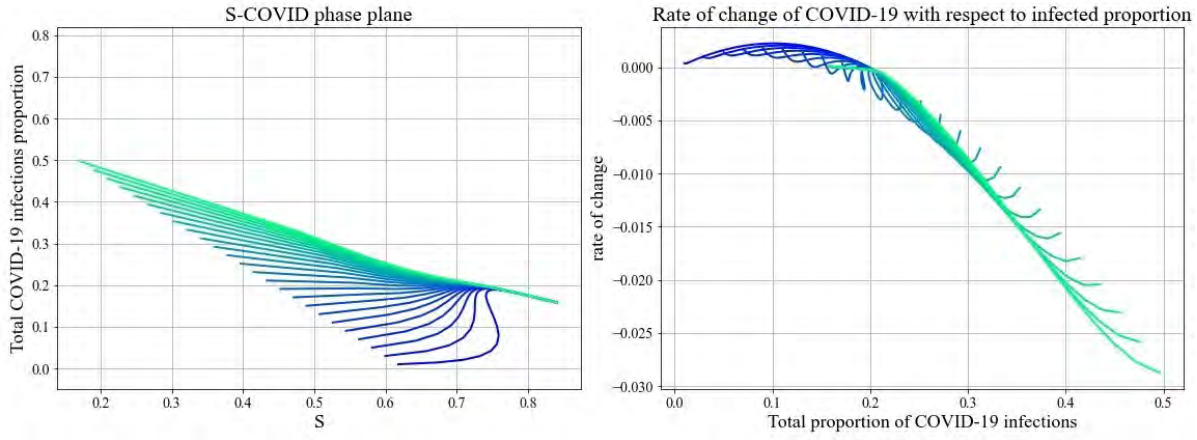


Figure 9: Plots of trajectories in the $S - I_C$ phase plane (left) and corresponding $\dot{I}_C$ vs. I_C trajectories (right) for decreasing values of $\mathcal{R}_C$.

(d) Phase plots for $\mathcal{R}_C = 1.4$



(e) Phase plots for $\mathcal{R}_C = 0.4$

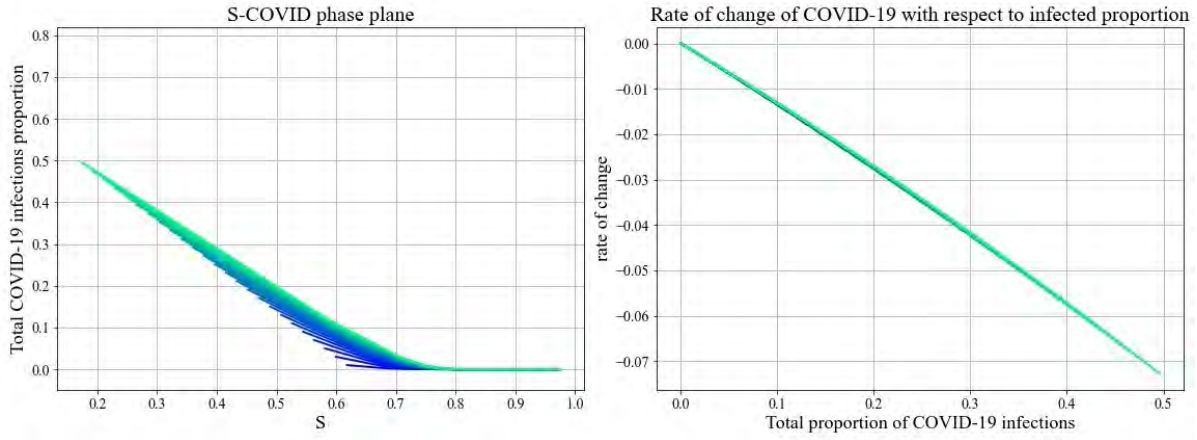


Figure 9: Plots of trajectories in the $S - I_C$ phase plane (left) and corresponding $\dot{I}_C$ vs. I_C trajectories (right) for decreasing values of $\mathcal{R}_C$ (with all values scaled by total population).

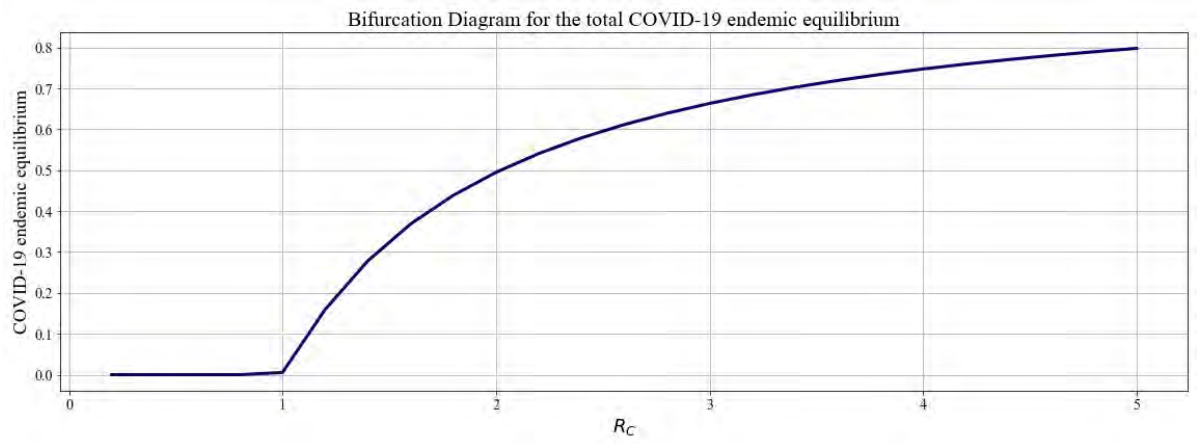


Figure 10: Bifurcation diagram for the COVID-19 fixed point's dependence on basic reproduction number $\mathcal{R}_C$, with the fixed point plotted as a proportion of the total population at equilibrium.

6 Discussions and Conclusions

The ongoing COVID-19 pandemic has far-reaching implications on health systems and management. Specifically, countries with high burdens of TB and HIV may experience particular challenges in coping with effects of possible interactions between infections which may aggravate one another immunologically. This paper presented an ODE system (4) modelling deterministic progression of interacting TB, HIV and COVID-19 infections by examining mutually exclusive compartments that represent infections and co-infections, an approach allowing for a dynamical systems analysis of the three epidemics. The model (4) has a biologically feasible solution space Ω, which is positively invariant with respect to the model.

An analysis of the TB-only, HIV-only, and TB/HIV submodels was performed to gain insights into the modes of action of individual infections and to explore the dynamics of TB and HIV in pre-COVID-19 times. We calculated independent basic reproduction numbers for TB and HIV using the NGM method. With the same approach, we found that the basic reproduction number in the TB/HIV interaction model $\mathcal{R}_{TH}$ is the maximum of the independent reproduction numbers $\mathcal{R}_T$ and $\mathcal{R}_H$. We proved that the IFE in both the TB-only and HIV-only sub-models is *locally asymptotically stable* for basic reproduction numbers less than unity, and unstable otherwise. We further showed that both individual infection subsystems satisfy conditions (H1) and (H2) in (16), and therefore proved that the IFE in each subsystem is *globally asymptotically stable* when it is locally asymptotically stable. Additionally, we showed the existence of a unique endemic equilibrium in the two subsystems when the basic reproduction number is greater than unity. We then investigated stability of the IFE in the TB/HIV sub-model with simulations using Runge-Kutta numerical integration to compute changes in compartments over time. The model was applied to Soweto, an urban complex in South Africa, using data from the region to allocate parameter values and taking Denmark's low transmission rates to investigate behaviour when the basic reproduction number is less than unity.

Under these conditions, the IFE appeared to be locally stable. We then attempted to use sub-model (33) to illustrate that a population in which one infection is already endemic provides an environment where it is easier for a second infection to appear and remain endemic when compared to an entirely infection-free environment. However, this exploration required complicated analyses, and was left for future work.

Finally, we performed an analysis on the full system (4) incorporating interactions between TB, HIV and COVID-19. We found that the basic reproduction number for the full model is the maximum of the individual infections' reproduction numbers. Again, the numerical scheme of the model was applied to Soweto. We explored two scenarios: first, low transmission rates of all three infections to investigate IFE stability; and second, actual Soweto transmission rates to investigate the effect of introducing COVID-19 into the pre-pandemic Soweto population.

The IFE was explored using Denmark's transmission rates. COVID-19's infection rate was introduced as 50 per 100 000 persons in the population and the simulation run for 100 years, allowing states to reach equilibrium. All infected compartments returned to IFE values, and the susceptible class tended towards total population. These results suggest that the full model's IFE is locally stable and attracting, but this is not a rigorous investigation and further examination or a mathematical proof is required to make conclusions regarding the IFE's stability.

Realistic transmission rates from Soweto were then used, resulting in $\mathcal{R}_T, \mathcal{R}_H, \mathcal{R}_C > 1$, and therefore $\mathcal{R}_{THC} > 1$. All HIV classes tended towards zero, whilst COVID-19 and TB compartments stabilised at non-zero proportions, implying that these infections remained endemic. Net effects of each infection were investigated; these indicated that total TB and total COVID-19 also stabilised, and could therefore be used to investigate infection dynamics in the phase space.

The COVID-19 endemic equilibrium point's dependence on the basic reproduction number was explored by plotting trajectories with different initial conditions in the COVID-19—S phase space for various values of $\mathcal{R}_C$. For each $\mathcal{R}_C > 1$, the system found a different endemic equilibrium, and the proportion of COVID-19 infections at equilibrium decreased with $\mathcal{R}_C$. Plots of COVID-19's rate of change against its proportion confirmed these endemic equilibria as fixed points. They appeared stable as all nearby trajectories tended towards them. For $\mathcal{R}_C < 1$, the system tended towards a COVID-19 free equilibrium. We created a bifurcation diagram (Figure 10) plotting the proportion of COVID-19 infections at equilibrium against $\mathcal{R}_C$, and found a bifurcation at $\mathcal{R}_C = 1$, as expected; for $\mathcal{R}_C < 1$, the system found the stable COVID-free point, but for larger values the system found increasing COVID-19 endemic equilibria.

These results are not necessarily representative of actual transmission dynamics in a system with TB, HIV and COVID-19; it is unlikely that HIV would be eradicated entirely under these conditions, as the simulations suggest. However, in the simulated scenario, COVID-19 remained endemic in a population initialised with only a small infected proportion. This indicates that, in scenarios where COVID-19 interacts with TB and HIV, it likely becomes difficult to manage, easily infecting the population and remaining endemic. The investigation of $\mathcal{R}_C = \frac{\beta_C}{\mathcal{R}_C}$ as a control parameter indicated that intervention methods should aim to decrease the $\mathcal{R}_C$ value to reduce the total proportion of COVID-19 in a population. This can be done by reducing transmission rate β_C, for example by encouraging physical distancing. Reducing $\mathcal{R}_C$ to below the threshold value 1 will

make control of the infection easier, potentially even allowing for eradication of COVID-19.

This paper has examined certain interactions between TB, HIV and COVID-19 using a compartmental model allowing for co-infections. We provided theoretical analyses of selected sub-models and gained insight into transmission dynamics, dependence on basic reproduction numbers and stability of the IFE. Numerical simulations of the full model allowed an analysis of dynamics of all three infections in a specific population. Further, phase space analysis allowed us to find a critical value for the control parameter $\mathcal{R}_C$ below which COVID-19 would theoretically be eradicated. Together, this has provided a first step towards addressing the thesis statement presented and enhanced understanding of these co-infections. Nevertheless, further investigation and verification would allow for more conclusive and applicable results.

6.1 Limitations

Using generalised deterministic compartmental systems to model infection interaction naturally relies on certain assumptions and simplifications that present limitations to the model and its applicability. Most importantly, it is difficult to produce reliable quantitative models or simulations when data are limited, as with the relatively new SARS-CoV-2. Ongoing research is providing novel findings regarding SARS-CoV-2 mortality, comorbidities, immunisation, and vaccination, thus models are restricted by current knowledge and data on the rapidly evolving virus [41, 80]. Other currently unknown data required to simulate the present model include the distribution and mean of TB's latent period [50], making selection of accurate parameter values regarding TB progression difficult.

Infections relying on physical contact between infectious individuals to spread, such as TB and COVID-19, do not necessarily affect a population homogeneously. Rather, they cluster, creating geospatial 'hotspots' of transmission [81, 82]. Cluster dynamics may play a role in spatial spread of infections, with overall projections that may be different from those forecasted by a homogeneous model [83]. System (4) models geographical space as homogeneous, but it may be relevant to include heterogeneous space or clustering in a similar co-infection model to simulate contact more realistically. The population itself is also assumed to be homogeneous and is not stratified by age, gender or other factors that may alter individual susceptibility and risks. This poses a limitation when examining infections that affect population groups differently [68].

Model (4) treats TB, HIV and COVID-19 similarly despite the former two being chronic and the latter acute. Provisions were made for this in the model by assuming that certain chronic infections or progressions do not occur during the short time period while an individual is infected with COVID-19. However, this simplification limited infection interactions as, in some cases, it excluded the possibility that co-infection may trigger disease progression. This assumption, coupled with the fact that COVID-19 recovery was not modelled, means it is possible for individuals to become consistently reinfected with COVID-19. This essentially means that, under the current model, these individuals are 'protected' against infection with other chronic illnesses because of their COVID-19 infection. Certain model transmission routes were included to reduce the likelihood of this occurring, such as the potential for infection with LTBI or HIV while COVID-19 infected. Specifically, because it is known that COVID-19 can catalyse progression from LTBI to TBD [14], this transmission path was incorporated. However, no similar information is known about HIV/COVID-19 interaction or disease progression currently, thus these transmissions were not modelled. It is plausible that this reduced number of transmission paths is the reason for the unlikely eradication of HIV while TB remained endemic.

Furthermore, [43] found that incorporating age of infection in models involving chronic and acute infection by using partial differential equations rather than ODEs changes stability criteria in a two-infection model. This is likely relevant in the present three-infection chronic/acute model as well, but age of infection is not included. Several relevant transmission paths of HIV are ignored to simplify the model, as are drug-resistant cases of TB. Further, it has been observed that COVID-19 infection and transmission patterns vary seasonally [44]; this model does not account for potential effects of such variations and does not model COVID-19 surges, long-COVID, or different virus mutations.

The results in this paper are based on local mathematical analyses of a model taking a simplified view of TB, HIV and COVID-19. However, as interactions between these infections are relatively novel, dynamics of co-infection are not well studied. There is little understanding of these interactions from a theoretical or mathematical perspective; thus, simplified models such as this remain valuable as a first exploration into infection interaction.

6.2 Future possibilities and concluding remarks

The analysis in this paper is limited to the submodels and two simulation scenarios. Future work should aim to complete the sub-model analysis for the addition of a new infection into a population where one infection is already endemic and extend this to the full model. Simulations presented in this paper are initialised near the start of the COVID-19 pandemic in March 2020. The pandemic has since developed significantly, and simulations initialised using current COVID-19 prevalence may provide more informative projections. Ongoing research is examining TB, HIV and

COVID-19 infections and their interactions (for example, [14]). As our epidemiological knowledge regarding effects of these interactions on infection progression and transmission increases, more of the modification parameters in this model can be included, which may have an impact on system dynamics. Currently, unknown or unexplored modification parameters are set to one in the simulations. An exploration of effects of different parameters on system dynamics could be conducted to determine which parameters are most important to target in an indirect attempt to reduce infection transmission and population prevalence. The direct inclusion of intervention methods in the model would also provide a means to evaluate possible future effects of interventions and how best to target interacting infections.

Other valuable model expansions include adding compartments to account for more infection states. Examples of this are adding compartments describing drug-resistant TB, ART, COVID-19 vaccination or immunity due to prior infection, different HIV viral loads, and compartments or modification parameters allowing for the differentiation between individuals who are treated or untreated for LTBI or TBD. Significantly, expanding the model to include transmission paths that are currently ignored, and COVID-19 immunity would likely prevent the eradication of HIV from the system, resulting in projections that are more representative of reality.

A detailed analysis of transmission, mortality and modification parameters applied to a variety of settings would be valuable to increase understanding of interactions in different locations and to prioritise effective mitigation strategies by region. Another interesting modification to the system is incorporating stresses on health care systems describing their ability to cope with increased demand for care when all three infections are present. Lower-middle income countries have limited hospital beds, intensive-care units, high-pressure oxygen machines and ventilation [41], all of which are required for treatment of severe COVID-19 cases. These counties often also experience high TB/HIV burdens, and treatment disruptions caused by the COVID-19 pandemic likely result in higher mortality risks [5]. On a more positive note, it is probable that advanced TB diagnostic tools in countries like South Africa can also be used to test for COVID-19, providing earlier and easier diagnoses [41]. This combination means the capacity of health care systems to respond to this triple infection burden may have significant impacts on region-specific dynamics. It may be beneficial to compare results of this spatially homogeneous deterministic model to results obtained from a stochastic model or one considering spatial effects. For example, modelling the same equations on a network describing physical contact would allow for the inclusion of the potential effects of geospatial transmission hotspots described above. This would allow for the evaluation of clusters where implementation of intervention methods may have the greatest impact.

It is important to note that model projections, here and otherwise, are not forecasts—trajectories are location-specific and can be changed with targeted interventions and treatment. However, this does not preclude the value of such research in increasing understanding of infection dynamics. Knowledge about the stability of simplified systems allows us to create targeted eradication strategies. This model, and others using similar methods, can be developed and analysed for stability without any quantitative data, as shown in Section 4. This allows for high-level behaviour analyses of data-deficient settings, which are particularly valuable when examining a novel virus such as SARS-CoV-2. Estimates of stability require less data than specifying a complete, simulated model [80], thus projections and simulations can be improved as data become available but are not data-dependent in the first place. Therefore, although simulation results in this paper are specific to Soweto, stability analyses and even qualitative dynamics are likely to extend to other regions in low- and middle-income countries in sub-Saharan Africa with a joint burden of TB and HIV during the COVID-19 pandemic.

Dynamical models such as the one presented in this paper provide us with insight into theoretical infection progression and transmission dynamics and allow us to develop and simulate intervention strategies. This research has illustrated the relevance of examining COVID-19 in conjunction with existing chronic epidemics to understand its impacts in different populations in both the short and the long term. Although this model is specific to the interactions between TB, HIV and COVID-19, it puts forward methods for analysing and forecasting interactions between multiple infections in general, which may prove useful both theoretically and in practice as new infection combinations appear globally.

Acknowledgments

I have received a great deal of support and assistance while writing this paper. First and foremost, thank you to my supervisor, Jan Bouwe—you were endlessly patient in explaining concepts and supportive of my sometimes-convoluted ideas. Thank you for your feedback and engagement, and your part in the final product. Thank you also to Mannus Schomaker for your excellent advice and encouragement. I'd like to acknowledge my tutor, Michael McAssey, for your quick and helpful responses to every problem, your willingness to scrutinise even seemingly minor issues and your continual guidance over the last three years. I am especially grateful to my parents for their endless support, and to friends and family for the stimulating discussions and happy distractions along the way. Finally, thank you to the InPrint editorial team for reading and re-reading this paper and for indulging my back-and-forth on almost everything. Your hard work is much appreciated!

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Sciences

To Infinity and Beyond: Visualization of Ordinal Numbers

A demonstrative case study of the epistemic role of visualization of mathematics within mathematical practice

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Abstract

Diagrams, sketches, graphs, and illustrations are examples of visualizations that can be found everywhere in mathematics. Yet, throughout the last three centuries, there have been constant shifts in attitude among mathematicians and philosophers of mathematics towards the role that visual thinking and visualization should play in mathematics, especially in rigorous formalism. Over the last few decades, there seems to be renewed interest in framing the role of visual thinking and visualization within different aspects of mathematical practice, but a consensus is yet to be formed. This research consists of a literature review examining the different attitudes developed towards visualization of mathematics, followed by a demonstrative case study concerning the integration of visualization in the context of ordinal numbers. This topic was chosen because it is often perceived as challenging to fully comprehend, due to its abstract theoretical nature. By presenting both a theoretical and an empirical analysis, this paper aims to contribute to the academic discussion, promote the integration of visualization in formal mathematics, and underpin its significance within various mathematical practices. The findings of the demonstrative case study did not point to a clear connection between such integration and successful acquisition of knowledge, but do indicate a link between participants' exposure to visual aids and the development of positive attitudes towards the studied material and the learning experience itself.

Keywords and phrases: *visualization, mathematics, ordinal numbers, visual thinking, mathematical practice, philosophy of mathematics, infinity*

Suggested citation:
Goren Y. (2022). To Infinity and Beyond: Visualization of Ordinal Numbers. A demonstrative case study of the epistemic role of visualization of mathematics within mathematical practice *AUC Undergraduate Journal of Liberal Arts and Sciences*, 19(1) pp. 36-62

imprint.

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1 Introduction

Visualization can be found everywhere in mathematics. Diagrams, sketches, and pictures are very often used by mathematicians in their daily work to gain a better understanding of abstract concepts, motivate claims, and support proofs in different sub-areas of mathematics. Moreover, it was shown that visual aid could increase comprehension among students, combat math phobia, and overall enrich the world of mathematical practice, research, and education [1, 2, 3, 4, 5, 6, 7, 8, 9]. Thus, the importance of visual thinking in mathematics is clearly evident.

A fitting example to demonstrate this is the following statement, also known as Nicomachus' Theorem:

$$\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2 \tag{1}$$

The sum of the first n cubed numbers equal to the squared sum of the first n numbers

At first glance, this statement seems somewhat counterintuitive. How could one be convinced it is indeed true? Different mathematicians have provided various methods to prove this theorem. One attempt, introduced by Charles Wheatstone in 1854 [10], is presented in the following proof sketch:

Proof.
We start by using the following identity which allows us to express n^3 as a sum of n consecutive odd numbers. [1]

$$n^3 = \underbrace{(n^2 - n + 1) + (n^2 - n + 1 + 2) + \ldots + (n^2 + n - 1)}_{n \text{ consecutive odd numbers}} \tag{2}$$

We then apply the above property in combination with the following well-known traits

$$\sum_{k=1}^{n} (2k - 1) = n^2 \tag{3}$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{n^2 + n}{2} \tag{4}$$

[1] This identity is based on properties of triangular numbers, which are numbers that represent the quantity of elements (e.g., dots) organized in an equilateral triangle (e.g., 3, 6, 21 are triangular numbers).

This yields the following deduction

$$\begin{aligned}\sum_{k=1}^{n} k^3 &= 1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 \\ &= 1 + 8 + 27 + 64 + \cdots + n^3\end{aligned}$$

By (2) we obtain

$$= \underbrace{1}_{1^3} + \underbrace{3+5}_{2^3} + \underbrace{7+9+11}_{3^3} + \underbrace{13+15+17+19}_{4^3} + \cdots + \underbrace{\left(n^2 - n + 1\right) + \cdots + \left(n^2 + n - 1\right)}_{n^3}$$

By (3) we obtain

$$= \underbrace{\underbrace{\underbrace{\underbrace{\underbrace{\underbrace{1}_{1^2} + 3}_{2^2} + 5}_{3^2} + 7}_{4^2} + 9}_{5^2} + 11 + \cdots + \left(n^2 + n - 1\right)}_{\left(\frac{n^2+n}{2}\right)^2}$$

The last step stemmed from the fact that we can express the number of odd numbers m in the series by adding one to the m^{th} number (k_m) and dividing the outcome by two as follows

$$m = \frac{k_m + 1}{2} = \frac{(n^2 + n - 1) + 1}{2} = \frac{n^2 + n}{2}$$

Hence

$$m^2 = \left(\frac{n^2 + n}{2}\right)^2.$$

Therefore,

$$= \left(\frac{n^2 + n}{2}\right)^2$$

By (4) we obtain

$$\begin{aligned}&= (1 + 2 + \cdots + n)^2 \\ &= \left(\sum_{k=1}^{n} k\right)^2\end{aligned}$$

Hence, we can conclude that

$$\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$$

as desired.

□

Even though this proof is considered relatively simple, it still requires the reader to concentrate and have some mathematical background to be able to understand it and be convinced about its validity. Moreover, it does not necessarily provide the reader with any intuitive understanding of the subject matter; one may comprehend these particular proof steps, but without the intuition, there is no guarantee that one is able to grasp the principles and fundamental ideas behind it. This hinders one's ability to obtain the means one will need to apply when encountering similar types of problems in the future.

Now, consider the following image:

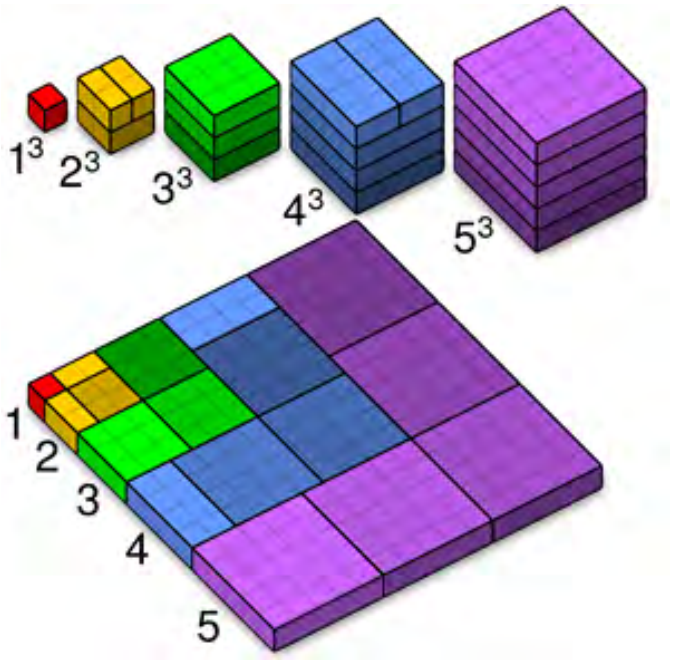


Figure 1: Visual verification that the sum of cubes is equal to the square of the sum. By courtesy of Wikipedia, the free encyclopedia. Created by CMG Lee [11].

Figure 1 provides an illustration that verifies Nicomachus' Theorem (1). It is simple and easy to understand, even without any mathematical background. This image on its own might not be considered a valid proof, as it only provides an instance in which the statement is true (i.e., the statement is true for $n \leq 5$, but does not provide any information for its validity for $n > 5$). Nevertheless, it complements the formal proof and clarifies its validity using a simple example (namely $n \leq 5$).

Finally, consider Figure 2. It provides yet another illustration of Nicomachus' Theorem. This illustration may still require the reader to have some basic mathematical understanding, but much less so than the knowledge required to comprehend the first, formal, proof. However, the second visualization maintains its generality, which the previous illustration lacks. Moreover, along with the short accompanying equations, one may regard this illustration as a valid rigorous visual proof of the statement.

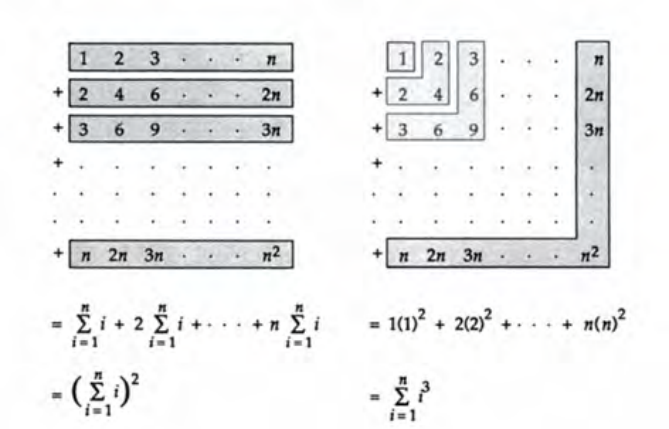


Figure 2: Visual proof that the sum of cubes is equal to the square of the sum. Created by Farhood Pouryoussefi [12].

Another example demonstrating the strength of visual thinking is the Dichotomy paradox, one of Zeno's four paradoxes that address counterintuitive properties of continuous time and space [13]. In essence, the paradox is intended to demonstrate that an object attempting to travel a finite distance would never reach its final destination. The statement is based on the idea that, in order to reach the endpoint, the object must first complete half of the course[2]. Since there are infinitely many halfway points, no moving entity can ever reach its destination in a finite amount of time.

Following Zeno's logic, one may conclude that the result of an infinite sum must always be infinite; indeed, this was the prevailing view around the 5th century BC [14]. However, years later, mathematicians resolved the paradox, realizing that summing infinitely many numbers can yield a finite result. This resolution can be demonstrated using infinite geometric series as a representation of the Dichotomy paradox. Let's consider the following statement:

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = 1 \tag{5}$$

The sum of the infinite geometric series $(\frac{1}{2})^n$ converges absolutely and will sum to a finite number, namely one.

This sum represents the distance S an object must travel in order to reach its destination, as seen from Zeno's viewpoint. Firstly, it reaches the halfway point of the final distance ($\frac{1}{2}S$). Then, it progresses halfway of the remaining path, i.e., a quarter ($\frac{1}{4}S$) of the original path, followed by a progress of half of the remaining path ($\frac{1}{8}S$), and so on.

[2] In order for one to travel some distance D, one must first travel halfway there ($D/2$), and before traveling halfway, one must travel one quarter ($D/4$) of the way, and so forth.

However, claiming that this sequence sums to one might seem intuitively wrong: how is it possible that the sum of infinite positive fractions gives a finite number?

To show this, let's consider the following proof:

Proof.

First, we define the sum

$$\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

in terms of the limit of the sum of the first n terms (denoted by s_n) as n approaches ∞

$$s_n = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots+\frac{1}{2^{n-1}}+\frac{1}{2^n}$$

Then, multiplying s_n by two results in

$$\begin{aligned} 2s_n &= \frac{2}{2}+\frac{2}{4}+\frac{2}{8}+\frac{2}{16}+\cdots+\frac{2}{2^n} \\ &= 1+\underbrace{\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^{n-1}}\right]}_{s_n-\frac{1}{2^n}} \\ &= 1+\left[s_n-\frac{1}{2^n}\right] \end{aligned}$$

Subtracting s_n from both sides yields

$$\begin{aligned} 2s_n - s_n &= 1+\left[s_n-\frac{1}{2^n}\right]-s_n \\ s_n &= 1-\frac{1}{2^n} \end{aligned}$$

Computing the limit of s_n as n approaches ∞ yields

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty}\left[1-\frac{1}{2^n}\right] = 1-\frac{1}{2^\infty} = 1-\frac{1}{\infty} = 1-0 = 1$$

And therefore we can conclude that

$$\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = s_n = 1$$

□

This simple proof sketch may convince the reader that indeed this infinite series sums up to one. However, as for the proof sketch presented for Nicomachus' Theorem, the above proof is convincing only for readers who have a sufficient mathematical background that allows them to follow and fully comprehend its steps.

Now, consider Figure 3, which illustrates the convergence of the infinite geometric sum (i.e., Statement 5). Even without a mathematical background, it is simple and straightforward to comprehend. Yet, similarly to Figure 1, one may claim that the above image cannot be considered as a rigorous proof on its own; rather, it provides the intuition towards constructing one.

Following this approach, by observing Figure 3, one can deduce an even stronger hypothesis; it is possible

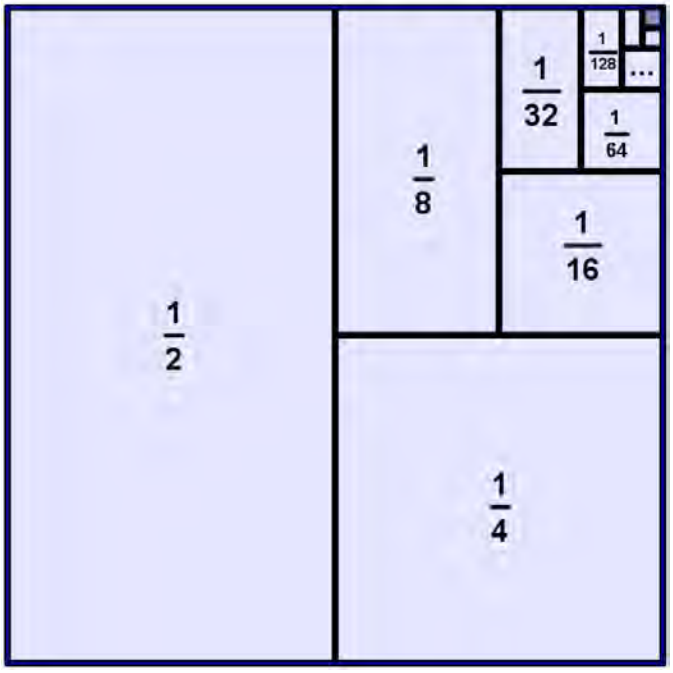


Figure 3: Visual verification that the infinite sum of the infinite sequence $(\frac{1}{2})^n$, as n approaches ∞, is equal to one. By courtesy of Patrick Honner [15].

to have infinitely many entities (that do not overlap with each other) in a finite space. Clearly, Figure 3 does not rigorously prove (nor is it attempting to prove) the latter, but it provides an intuition from which one may construct a formal proof of the properties emerging from the visual representation.

Thus, visualization can act not only as a rigorous, complete proof, as demonstrated in Figure 2, but also as a means of discovery, revealing properties that otherwise would be burdensome to disclose and helping to guide the way towards formulation of rigorous mathematical proofs. In spite of the many advantages of visualization of mathematics demonstrated above, and the broad use of visual thinking in the daily work of mathematicians, when it comes to mathematical formalism, the lack of visualization is evident [16]. This research seeks to challenge this norm and aims to lay out the advantages of using visualization in mathematics (with a focus on set theory) and to demonstrate the legitimate, formal role it (should) play. This is done by analyzing different approaches developed towards the role of visualization of mathematics within mathematical practice and by examining the effects of integrating visual aids within mathematical reasoning via an empirical study. In essence, this paper strives to answer the following research question:

What is the epistemic value of visualization of mathematics within mathematical practice of ordinal numbers?

2 Methodology

In order to answer the research question and test the hypothesis regarding the role of visualization within mathematical practice, this paper is divided into two main parts. The first part (Sections 3 and 4) entails a literature review in which the different attitudes developed towards visualization of mathematics throughout the last three centuries are examined. The second part (Sections 5 and 6) aims to test the theory in practice, and test the hypotheses and theories identified in the former part of the paper. This is done using a questionnaire concerning ordinal numbers, a sub-field of set theory, chosen as a demonstrative case study.

The scope of the literature review encompasses visualization of mathematics on a broader spectrum, in which the dominant historical and contemporary approaches towards the role of visualization within mathematical practice are deduced via qualitative analysis. This provides a better understanding of the prominent views on the subject matter and their origins, while evaluating the various advantages of integrating visualization within mathematics.

The second part of the paper applies the conclusions stemming from the historical overview by conducting a qualitative and quantitative study around ordinal numbers and related mathematical concepts. This part also includes an introduction to ordinal numbers (Section 5), which presents the relevant mathematical concepts and proofs on which the questionnaire and the accompanied test are based. The survey is designed to evaluate the extent to which incorporating visual aids affects participants' comprehension of ordinal numbers when they are introduced to the subject matter. A detailed description of the survey, as well as its design and the motivation behind it, are presented in Section 6.

By combining these two methodologies, this paper aspires to promote the integration of visualization in formal mathematics, underpin its significant epistemic role within mathematical practice, and recommend its use as a practical tool to combat math phobia. Moreover, it could help to bridge the gap between philosophy of mathematics and mathematical practice, and potentially accelerate mathematical research.

3 Historical Development

During the late 18th century, the prevailing view among philosophers and mathematicians was that visual thinking and visual intuition are vital to mathematical knowledge [6, 17]. As proclaimed by the German philosopher Immanuel Kant: *"mathematics can achieve nothing by concepts alone but hastens at once to intuition"*[3] [17].

[3]The German word *'Anschauung'*, which is used to describe visual imagination and perception, is translated as *'intuition'* in this context [18].

However, the zeitgeist of the late 19th century shifted away from this view, and the reliability of visual thinking was heavily questioned. The principal cause for the shift was that spatial intuition was often found to be wrong and completely deceptive under closer inspection [4].

There are two commonly used examples in this context. Firstly, the notion that any continuous function must be differentiable everywhere except at isolated points. Secondly, the implicit presumption in Euclid's *Elements 1.1* that the two circumferences drawn for the construction of an equilateral triangle over any given fragment intersect at the vertex of the equilateral triangle. In both instances, the assertion appears evident based on its visual illustration, but was later found to be unjustified. In the former example, the fallacy was uncovered following Weierstrass's discovery of continuous but nowhere differentiable functions. In the second example, the uncovering of the mistaken belief was the consequence of the comprehension that one can only ensure the existence of an intersection point between two circumferences by accepting the continuity axiom [7].

These revelations, alongside others, invalidated previously accepted beliefs that relied on these fallacious assumptions. This led to a lack of unified stance within the mathematics community regarding the credibility of visual thinking, a dispute later described by Hans Hahn as *"the crisis in intuition"* [19]. This influenced many mathematicians in the late 19th century and early 20th century to turn away from visual reasoning towards a stricter attitude regarding acceptable thinking in mathematics, in which only rigorous reformulation, articulated using formulas and text, had an epistemic validity, while visualization did not [20]. In the words of the mathematician Moritz Pasch, *"...the theorem is only truly demonstrated if the proof is completely independent of the figure"* [21]; or, as stated by the mathematician Bertrand Russell, *"In the best books there are no figures at all"* [20]. This approach is also manifested in Edmund Landau's image-free calculus textbook [22]. Nevertheless, visual thinking as a peripheral tool to ease comprehension was still acceptable, despite its lost legitimacy as a means of justification [4, 7]. This view is evident in Hilbert's approach towards visualization: while he believed that *"a theorem is only proved when the proof is completely independent of the diagram"*, he also acknowledged that *"visual perception still plays the leading role in geometry"* [23].

Rooted in the foundational controversies mentioned above, and in accordance with the emerging stricter approach among mathematicians, many philosophers of mathematics in the early 20th century adopted a simplified view of mathematics. This view focused almost exclusively on questions regarding truth and proof, focusing the philosophical discussion on the validity of self-evident axioms, the consistency of proofs, and the soundness of theorems stemming from the former [8]. The excessive emphasis on ontological questions led

mainstream philosophy of mathematics to stick to a very narrow view of mathematical epistemology. Consequently, much of the richness of day-to-day mathematical practice, which falls outside this framework, was disregarded. Thereby, a chasm has opened between theoretical philosophy of mathematics and mathematical practice regarding visualization of mathematics. Consequently, this prevented the creation of a unified (embracing) attitude towards the formal role it should play within mathematical practice.

In the late 20th century, the distaste for visual thinking diminished, and interest in the different roles of visualization within mathematics re-emerged [4, 6]. Significant reasons for this include the new visualization techniques developed within computer science, which have greatly influenced mathematics. In particular, the fields of geometry and chaos theory have especially underlined the potent role of visualization within mathematical practice [24, 25, 26]. These new computer-generated graphics allowed researchers to exhibit information in diverse visual formats, which enabled new, previously unattainable and highly complex discoveries through rapid visual comprehension. For instance, this is illustrated in fractal theory, a sub-field of chaos theory, where the revelation that the Julia sets are contained within the Mandelbrot set would have been impossible to discern analytically without the visual aid created by the computer. Figure 4 presents a computer-generated illustration of the two sets. In the context of geometry, Hoffman [25] highlighted the significance of computer-generated visuals and their contribution to the discovery of new minimal surfaces, describing them as *"part of the process of doing mathematics"*. Along the same lines, Palais [26] emphasized that *"the visualizations actually helped point the way to rigorous mathematical proofs"*.

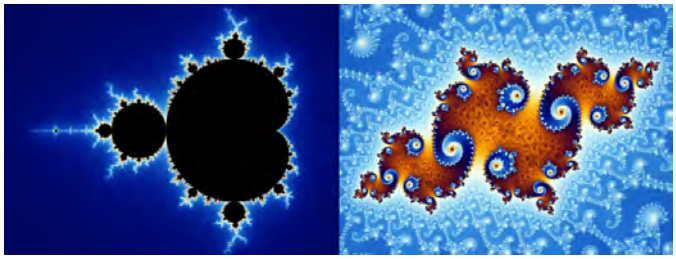


Figure 4: Computer-generated visualization of the Mandelbrot set (left-side image) and Julia sets (right-side image). The latter was generated by 'zooming' into the boundary of the former. By courtesy of Wikipedia, the free encyclopedia. Created by Wolfgang Beyer with the program Ultra Fractal 3 [27].

In their work, Fomenko [28] and Needham [29] identified and underlined the significant effects of computer science on the ongoing view towards the increased use of visual methods within mathematical practice. Both scholars independently stressed that visualization has heuristic and educational benefits. Fomenko illustrated that visualization can provide an instantaneous introduction to mathematical concepts, claiming that

> *"it happens rather frequently that the proof of one or another mathematical fact can at first be 'seen', and only after that (and following the visual idea) can we present a logically consistent formulation, which is sometimes a very difficult task requiring serious intellectual efforts"* [28].

However, neither Fomenko nor Needham consider visualization a means of justification, or rigorous self-sufficient proof. Even Nelsen, the editor of the book *Proofs Without Words: Exercises in Visual Thinking*, claimed that *"proofs without words are not really proofs"* [12]. While Needham acknowledged that many of the arguments presented in his book *"are not rigorous, at least as they stand"*, he underscored that

> *"an initial lack of rigor is a small price to pay if it allows the reader to see into this world more directly and pleasurably than would otherwise be possible"* [29].

Nevertheless, not everyone shares this view, and, consequently, the epistemic role of visualization is still a focal subject for debate [4]. Over the last few decades, mathematicians and philosophers of mathematics have embraced different perspectives towards visualization. According to one extreme view, a stand-alone image can be regarded as valid proof. In their work, Allwein and Barwise [30], Brown [31], and Jamnik [32] supported this view and presented their attempts to show the efficiency of visualization as a method of proof. The other extreme of the spectrum supports the predominant view of the 19th century, claiming that visualization has no place in proofs at all (e.g., [33]). Between these extremes, there are scholars such as Fomneko and Needham, who believe that although no stand-alone illustration can be quantified as a proof, visualization can play an epistemic role in reasoning that composes a proof, beyond the purely heuristic role [4, 34, 35]. The educational advantages stemming from the integration of visual thinking within mathematical education is also explored and demonstrated by mathematical educators, such as Arcavi [1], Clements [2], Hughes-Hallett [36], Nardi [37], and Zimmermann and Cunningham [38], all of whom found the implementation of visualization beneficial to students' grasp of mathematics.

From research to education, visual thinking is ubiquitous in mathematics, and its utility is undeniable. However, as demonstrated above, there is no clear consensus regarding the formal role of visualization within mathematics. As such, the magnitude of its effect is often subject to one's discretion. Nevertheless, in recent years a general paradigm shift seems to have occurred; over the past few decades, philosophers of mathematics have acquired a renewed (albeit still relatively narrow)

interest in mathematical practice [39]. Consequently, more and more philosophers and mathematicians are coming to the conclusion that visualization of mathematics could and should play a bigger role in mathematical practice [3, 4, 6, 16, 40, 41, 42], suggesting that

> *"visual forms of representation can be important, not just as heuristic and pedagogical tools, but as legitimate elements of mathematical proofs"* [34].

To support the positive change in approach towards visualization of mathematics, the following sections of this paper attempt to empirically demonstrate the beneficial effects of integrating visuals as a means of reasoning within mathematical proofs (Section 4) and as a means of gaining knowledge when encountering a new mathematical topic (Section 6).

4 Infinity and the Diagonal Argument

"To infinity and beyond"
(Buzz Lightyear [43])

According to the mathematical definition, *infinity* is *"the assumed limit of a sequence, series, etc., that increases without bound"* [44]. In simpler words, an infinite object is endless, the 'biggest' entity (i.e., bigger than all counting numbers) possible. Moreover, if an object has no bounds, it is implied that we could never reach its endpoint. So how is it possible to have anything bigger than it? At first glance, this seems mathematically impossible. Does it imply that Buzz Lightyear's catchphrase is as fictitious as his character? What did he mean when he said, *"To infinity and beyond"*? How can one grasp the full meaning of an abstract concept like infinity, not to mention the idea that there exists an entity that is beyond infinity? And if so, what could this entity be?

Throughout history, mathematicians have developed different intuitions towards explaining the concept of infinity. The Greek philosopher Aristotle, for example, postulated that there are two types of infinities—an *actual infinity* and a *potential infinity*. He defined *actual infinity* as a definitive and comprehensive infinity that entails an infinite number of elements. Conversely, *potential infinity* is never complete, such that *"one thing is always being taken after another, and each thing that is taken is always finite, but always different"* [45]. Aristotle believed the former is beyond the bounds of possibility, as its existence contradicts the belief that there is nothing greater than God's power. Indeed, this was the prevailing view among mathematicians and philosophers until the 19th century. The profound shift in approach was largely influenced by the work of the Russian-born mathematician Georg Cantor, who asserted that

> *"the actual infinite arises in three contexts: first when it is realized in the most complete form, in a fully independent otherworldly being, in Deo, where I call it the Absolute Infinite or simply Absolute; second when it occurs in the contingent, created world; third when the mind grasps it in abstracto as a mathematical magnitude, number or order type"* [46].

In 1874, under this interpretation of infinity, Cantor was the first to show, using a rigorous formal proof, that infinity comes in different sizes [47]. More specifically, Cantor showed that it is not possible to construct a one-to-one (i.e., injective) mapping from the real numbers R to the natural numbers N, i.e., he demonstrated that the (infinite) set of all real numbers R is strictly bigger than the (infinite) set of all natural numbers N. Years later, in 1891, Cantor published an alternative, simpler method to prove his theorem, which is famously known as *Cantor's diagonal argument* [48]. Due to its (relative) simplicity, analogues of the argument are commonly employed by mathematicians to establish the existence (or non-existence) of different objects (e.g., Gödel's first incompleteness theorem, the halting problem) [49]. Cantor's diagonal argument is as follows:

Theorem 1 (**Cantor's diagonal argument**)**.** *$\mathbb{R}$ is uncountable (i.e., there is no bijection between the infinite set $\mathbb{R}$ and the infinite set $\mathbb{N}$, meaning $\left|\mathbb{R}\right| \neq \left|\mathbb{N}\right|$).*

Proof. By contradiction.

We start by defining a set $\mathcal{T}$ as the set of all infinite sequences of binary digits (i.e., zeros and ones) and assume this set is countable[4].
Therefore, we can form an enumeration $s_1, s_2, s_3, \ldots, s_n$ of all the elements of $\mathcal{T}$, as illustrated in the following diagram,

$$\begin{array}{lcccccccccc}
s_1 & = & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
s_2 & = & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots \\
s_3 & = & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & \cdots \\
s_4 & = & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & \cdots \\
s_5 & = & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & \cdots \\
s_6 & = & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & \cdots \\
s_7 & = & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\
s_8 & = & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & \cdots \\
\vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}$$

such that each sequence in $\mathcal{T}$ is represented by its corresponding enumeration, namely s_n (for some $n \in \mathbb{N}$).
Now, we construct a sequence t, such that for each n, the n^{th} digit of t is determined by the following function:

$$t_n = \left\{ \begin{array}{ll} 0 & \text{if } s_{nn} = 1 \\ 1 & \text{if } s_{nn} = 0 \end{array} \right\} \tag{6}$$

[4] There exists a bijection between the set $\mathcal{T}$ and the set $\mathbb{N}$.

Hence, each digit of t is defined as complementary to s_{nn} (i.e., the n^{th} digit in the expansion of s_n), as illustrated in the following diagram,

$$\begin{array}{lcccccccc}
s_1 & = & \mathbf{0} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
s_2 & = & 1 & \mathbf{1} & 1 & 1 & 1 & 1 & 1 & \cdots \\
s_3 & = & 0 & 1 & \mathbf{0} & 1 & 0 & 1 & 0 & \cdots \\
s_4 & = & 1 & 0 & 1 & \mathbf{0} & 1 & 0 & 1 & \cdots \\
s_5 & = & 1 & 1 & 0 & 1 & \mathbf{0} & 1 & 1 & \cdots \\
s_6 & = & 0 & 0 & 1 & 1 & 0 & \mathbf{1} & 1 & \cdots \\
s_7 & = & 1 & 0 & 0 & 0 & 1 & 0 & \mathbf{0} & \cdots \\
s_8 & = & 0 & 0 & 1 & 1 & 0 & 0 & 1 & \mathbf{1} \cdots \\
\vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \ddots
\end{array}$$

such that we obtain

$$t = \mathbf{1}\ \mathbf{0}\ \mathbf{1}\ \mathbf{1}\ \mathbf{1}\ \mathbf{0}\ \mathbf{1}\ \mathbf{0} \cdots .$$

As t is an infinite sequence of binary digits, it must be in the set $\mathcal{T}$ by definition. Hence, by assumption, it must appear in the enumeration. Without loss of generality, assume there exists some $i \in \mathbb{N}$ such that

$$t = s_i$$

Then

$$t_i = s_{ii}$$

But from (6) we obtain

$$t_i \neq s_{ii}$$

Then, it follows that

$$t \neq s_i$$

contradiction.

$\Rightarrow \mathcal{T}$ is uncountable.

Now, to show that $\mathbb{R}$ is also uncountable, we can construct an injective function

$$f : \mathcal{T} \longrightarrow \mathbb{R}$$

such that each binary string $t \in \mathcal{T}$ is mapped to the corresponding decimal fraction $0.t \in \mathbb{R}$

$$t \mapsto 0.t$$

f is clearly injective, as it maps each string to a unique decimal fraction.
Therefore, we can conclude that

$$Image(f) \subseteq \mathbb{R}$$

Since $\mathcal{T}$ is uncountable, $Im(f)$ is uncountable. Hence, $\mathbb{R}$ must be uncountable.

□

As required, Cantor's diagonal argument verifies the existence of infinities of different sizes. The rationale behind the diagonal argument can be convoluted when presented without any visual aid, and therefore it is often accompanied by a similar diagram as the one presented in the proof above. (For the skeptical readers, imagine encountering the diagonal argument without the accompanying diagrams; you might claim it does not make the argument more challenging to follow, but I highly doubt you will argue that eliminating visual illustrations makes it easier).

Thus, using visual aids makes the diagonal argument more accessible and easier to grasp intuitively, which is especially useful when applied to elements of infinite sets, concepts that are hard to grasp to begin with. More importantly, it confirms that Buzz Lightyear knew what he was talking about.

Now, after successfully concluding that infinity can come in different sizes, it is only logical to wonder whether other infinite sets exist. If so, how can we describe, compare, and order them? Luckily, these types of questions also preoccupied Georg Cantor, which led him to the formulation of *cardinal* and *ordinal numbers* [50]. Due to the scope of the paper, the former, unfortunately, will not be discussed further. The latter, however, is presented in detail in the following section.

5 Introduction to Ordinal Numbers

Note for the reader: a basic understanding of mathematical concepts is assumed. For more detailed descriptions and definitions please see [51].

All formal definitions and theorems presented in this section were taken and adapted from [51].

An *ordinal number* (or '*ordinal*' for short) describes the numerical position or order of an object with respect to other objects. Mathematically speaking, ordinals can also be seen as a generalization of the set-theoretic representation of the natural numbers. Therefore, it is essential to construct the natural numbers within the framework of set theory (i.e., as sets) before delving into the world of ordinal numbers.

5.1 Natural Numbers as Sets

Intuitively speaking, the natural numbers (0, 1, 2, 3, 4, 5, ...) are well known to all. As such, we can readily provide examples of sets of natural numbers containing zero, one or two elements:

The set $\{\}$, usually denoted by $\emptyset$, is a set of zero elements

The set $\{a\}$, for any a, is a set of one element (e.g., $\{2\}$, $\{5\}$)

The set $\{a, b\}$, for any pair of distinct a and b, is a set of two elements (e.g., $\{2, 5\}$)

However, in order to represent the natural numbers as sets in a consistent manner, we need to define a unique, coherent representation for each natural number. For instance, out of all sets containing a single element, we ought to select one set to represent the number 1. Similarly, such a set should be selected for all other natural numbers.

Let's start with the first natural number - *zero*. Since there is only one set that contains no elements, namely $\emptyset$, we choose it as a representative. Hence, we define

$$0 = \emptyset.$$

Now, we continue to the next number - *one*. We need to choose a representative out of all sets having one element. Unlike the former, in this case we have infinitely many such sets to choose from. However, as we have already defined one specific object, viz. 0, the logical choice would be $\{0\}$. Thus, we define

$$1 = \{0\} = \{\emptyset\}.$$

Following the same idea, we can construct the two-element set, representing the number two, by choosing the objects we just defined, viz. 0 and 1 as its two elements, such that we observe

$$2 = \{0, 1\} = \{\emptyset, \{\emptyset\}\}.$$

Similarly, we can carry on with the process and construct

$$3 = \{0, 1, 2\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$$
$$4 = \{0, 1, 2, 3\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}\}$$
$$5 = \{0, 1, 2, 3, 4\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}\}\}$$

and so on . . .

As follows, we can represent any natural number $n \in \mathbb{N}$ as the set containing all the natural numbers that are smaller than n, such that we obtain a set that contains precisely n elements:

$$n = \{0, 1, \ldots, n-1\} = \{m \in \mathbb{N} \mid m < n\}.$$

Now, let's revise the construction of the numbers presented above to obtain the following:

$$1 = \{0\} = \{\} \cup \{0\} = 0 \cup \{0\}$$
$$2 = \{0, 1\} = \{0\} \cup \{1\} = 1 \cup \{1\}$$
$$3 = \{0, 1, 2\} = \{0, 1\} \cup \{2\} = 2 \cup \{2\}$$
$$4 = \{0, 1, 2, 3\} = \{0, 1, 2\} \cup \{3\} = 3 \cup \{3\}$$
$$5 = \{0, 1, 2, 3, 4\} = \{0, 1, 2, 4\} \cup \{4\} = 4 \cup \{4\}$$

As one may notice, each of the natural numbers presented above can be expressed as the set obtained by adjoining the preceding number to its set representation. More generally, given a natural number n, one can obtain the *succeeding* natural number, namely $n+1$, by adjoining the element n to the set n. Formally, we define this operation as follows:

Definition 1 (**Successor**)**.** The successor of a set x is the set $\mathcal{S}(x) = x \cup \{x\}$

Hence, we say $n+1$ is the *successor* of n, i.e., $\mathcal{S}(n) = n+1$. Moreover, if n is a natural number, it follows that $n+1$ is also a natural number.

5.2 Ordinal Numbers

As mentioned before, we can think of ordinals as a generalization of the natural numbers; just like every natural number n is identified with the set of all smaller natural numbers, an ordinal number is identified with the set that contains all its predecessors but does not contain itself. Hence, all natural numbers are ordinal numbers.

By analogy, the successor operation defined above for natural numbers can be generalized to ordinal numbers. This would allow us to continue the counting process beyond the natural numbers, which is where things start to get interesting.

Cantor was the first to coin the transfinite number *omega*, denoted by the Greek letter ω, which is defined as the first transfinite number that comes after all the natural numbers; hence, it is identified with the set $\mathbb{N}$ of all natural numbers. A visual notation of ω can be seen in Figure 5.

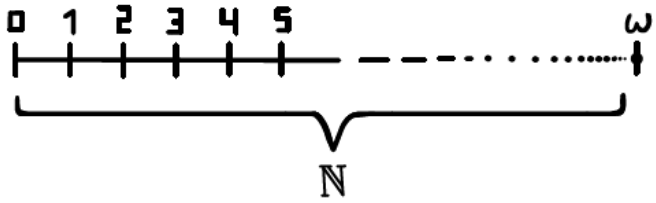


Figure 5: $\omega = \mathbb{N} = \{0, 1, 2, 3, \ldots\}$

With the notion of ω, ordinals allow us to continue the process of counting beyond the natural numbers, as they allow us to describe and compare positions of infinite entities (again, reassuring that Buzz Lightyear was clearly familiar with ordinal numbers). Using ω and the successor operation, we can easily define the next transfinite ordinals as follows:

$$\mathcal{S}(\omega) = \omega \cup \{\omega\} = \{0, 1, 2, 3, \ldots, \omega\} = \omega + 1$$
$$\mathcal{S}(\mathcal{S}(\omega)) = (\omega) \cup \{(\omega)\} = \{0, 1, 2, 3, \ldots, \omega, (\omega)\}$$
$$= \{0, 1, 2, 3, \ldots \omega, \omega + 1\} = (\omega + 1) + 1 = \omega + 2$$

and so forth . . .

This is illustrated visually in Figure 6.

Intuitively, one can assume that the successor of an ordinal number is also an ordinal number. We say α is a *successor ordinal* if and only if there exists an ordinal β such that $\alpha = \beta + 1$. Otherwise, we call α a *limit ordinal*. For example, 0 is clearly a *limit ordinal*, as within the set $\mathbb{N}$ it has no predecessors and therefore, there is no

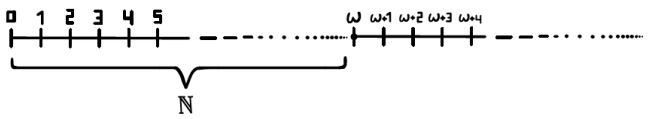


Figure 6: Ordinals allow us to continue the counting process beyond $\mathbb{N}$

ordinal β such that $0 = \beta + 1$. Similarly, we say ω is the *first transfinite limit ordinal*, as it is not a successor of any ordinal. Formally, we define ordinals as follows:

Definition 2 (**Ordinal Numbers**)**.** A set α is an ordinal number if:

1. α is transitive (i.e., every element of α is also a subset of α), and
2. α is well-ordered by $\in_\alpha$ (i.e., $(\alpha, \in)$ is linearly ordered and every nonempty subset of α has a least element).

From this definition, it is easy to see why the natural numbers are ordinals, as they are transitive and well-ordered by $\in$. Moreover, it ensures that if α is an ordinal, then $\mathcal{S}(\alpha)$ is also an ordinal, and every element of an ordinal number is an ordinal number itself.

5.3 Ordinal Arithmetic

We define addition and multiplication of ordinals recursively as follows:

Definition 3 (**Addition of Ordinals**)**.** For all ordinal numbers β,

1. $\beta + 0 = \beta$,
2. $\beta + (\alpha + 1) = (\beta + \alpha) + 1$ for all α,
3. $\beta + \alpha = \sup\{\beta + \gamma \mid \gamma < \alpha\}$ for all limit $\alpha \neq 0$.

The intuition behind addition of ordinal numbers β and α, namely $\beta+\alpha$, is that α is 'attached' after β. The result obtained is the ordinal number that has the same length (or order type) as the expression $\beta + \alpha$. See Figure 7 for a visualization of the addition operation.

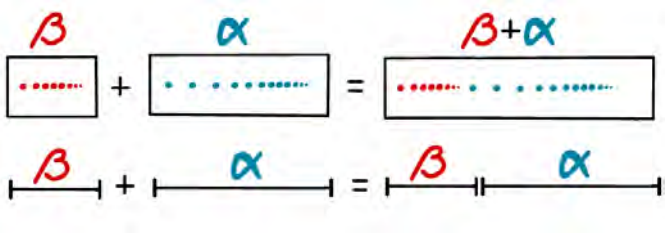


Figure 7: Visual representation of addition of ordinals

Definition 4 (**Multiplication of Ordinals**)**.** For all ordinal numbers β:

1. $\beta \cdot 0 = 0$
2. $\beta \cdot (\alpha + 1) = \beta \cdot \alpha + \beta$ for all α
3. $\beta \cdot \alpha = \sup\{\beta \cdot \gamma \mid \gamma < \alpha\}$ for all limit $\alpha \neq 0$

The intuition behind multiplication of ordinal numbers α and β, namely $\alpha \cdot \beta$, is that α is 'attached' after α, which in turn is 'attached' after α, which is 'attached' to another copy of α... β many times. In other words, we add together β copies of α. The result obtained is the ordinal number that has the same length (or order type) as the expression $\beta \cdot \alpha$. See Figure 8 for a visualization of the multiplication operation.

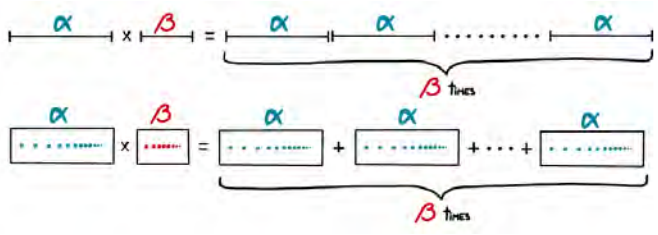


Figure 8: Visual representation of multiplication of ordinals

As for natural numbers, addition and multiplication of ordinals is associative[5]: for any ordinals $\alpha,\ \beta,\ \gamma$, the properties $\alpha+(\beta+\gamma) = (\alpha+\beta)+\gamma$ and $\alpha\cdot(\beta\cdot\gamma) = (\alpha\cdot\beta)\cdot\gamma$ hold.

Unlike natural numbers, addition and multiplication of ordinals numbers is not commutative[6]: the properties $\alpha+\beta = \beta+\alpha$ and $\alpha\cdot\beta = \beta\cdot\alpha$ do not hold for all pairs of ordinals $\alpha,\ \beta$.

This is demonstrated in the following examples.

Example 1: $\omega + 3 \neq 3 + \omega$

- $\omega + 3 := \underbrace{\{0, 1, 2, 3, \ldots\}}_{\omega} + \underbrace{\{0, 1, 2\}}_{3}$

By 'attaching' 3 (the second ordinal) to ω (the first ordinal), we get

$$= \underbrace{\{0, 1, 2, 3, \ldots, 0', 1', 2'\}}_{\omega+3}$$

and by relabeling the elements of the second ordinal 3, we obtain

$$= \underbrace{\{0, 1, 2, 3, \ldots,}_{\omega} \underbrace{\omega, \omega+1, \omega+2\}}_{3} = \omega + 3\ .$$

[5]Associativity: The outcome will not change if we rearrange the parenthesis in the argument.

[6]Commutativity: The outcome will not change if we rearrange the order of the elements in the argument.

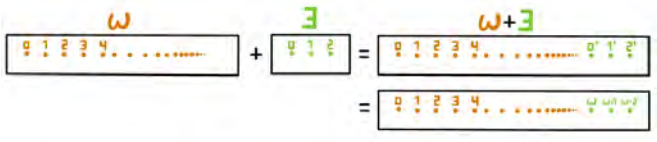


Figure 9: Visual representation of $\omega + 3$

- $3 + \omega := \underbrace{\{0,1,2\}}_{3} + \underbrace{\{0,1,2,3,\ldots\}}_{\omega}$

By 'attaching' ω (now the second ordinal) to 3 (now the first ordinal), we get

$$= \underbrace{\{0,1,2,0',1',2',3',\ldots\}}_{3+\omega}$$

and by relabeling the elements of the second ordinal ω, we simply obtain

$$= \underbrace{\{0,1,2}_{3}, \underbrace{3,4,5,6,7,\ldots\}}_{\omega} = \omega\,.$$

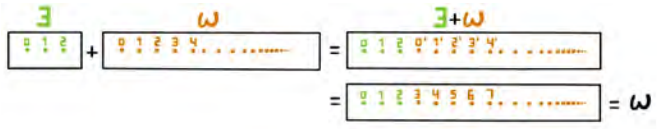


Figure 10: Visual representation of $3 + \omega$

We notice that $\omega + 3$ is clearly different from ω (i.e., $\omega + 3 \neq \omega$) because $\omega + 2$, for example, is an element of the set $\omega+3$, but is not an element of ω. However, $3+\omega$ is just ω (i.e., $3 + \omega = \omega$). This is because, as shown above, we can relabel the elements of $3 + \omega$, in such a way that it will just look like ω itself. Therefore, we can conclude that $\omega + 3 \neq 3 + \omega$ and, consequently, that addition of ordinals is not commutative. Visual descriptions of $\omega + 3$ and $3 + \omega$ can be found in Figures 9 and 10, respectively.

Example 2: $\omega \cdot 2 \neq 2 \cdot \omega$

- $\omega \cdot 2 := \underbrace{\{0,1,2,3,\ldots\}}_{\omega} \cdot 2$

Hence, we 'attach' together two copies of ω, one after the other

$$= \underbrace{\{0,1,2,3,\ldots\}}_{\omega} + \underbrace{\{0,1,2,3,\ldots\}}_{\omega}$$
$$= \underbrace{\{0,1,2,3,\ldots,0',1',2',3',\ldots\}}_{\omega\cdot 2}$$

and by relabeling the elements of the second copy of ω, we obtain

$$= \underbrace{\{0,1,2,3,\ldots}_{\omega}, \underbrace{\omega,\omega+1,\omega+2,\omega+3,\ldots\}}_{\omega}$$
$$= \omega + \omega$$

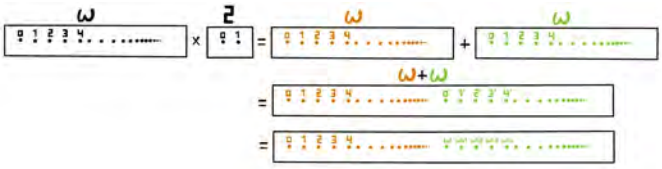


Figure 11: Visual representation of $\omega \cdot 2$

- $2 \cdot \omega := \underbrace{\{0,1\}}_{2} \cdot \omega$

We 'attach' together omega copies of 2, one after the other

$$= \underbrace{\underbrace{\{0,1\}}_{2} + \underbrace{\{0,1\}}_{2} + \underbrace{\{0,1\}}_{2} + \underbrace{\{0,1\}}_{2} + \ldots}_{\omega \text{ many times}}$$
$$= \underbrace{\{0,1,0',1',0'',1'',0''',1''',\ldots\}}_{2\cdot\omega}$$

and by relabeling the elements of $2 \cdot \omega$, we simply obtain

$$= \underbrace{\{0,1}_{2}, \underbrace{2,3}_{2}, \underbrace{4,5}_{2}, \underbrace{6,7}_{2}, \underbrace{\ldots\}}_{\text{etc.}} = \omega$$

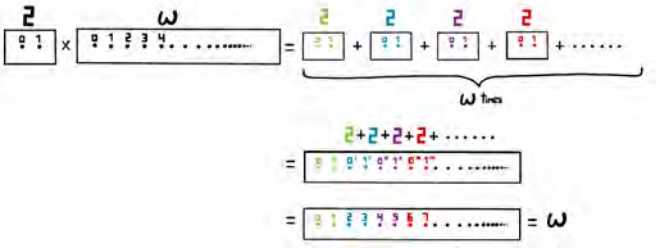


Figure 12: Visual representation of $2 \cdot \omega$

Just like the set $3 + \omega$, after relabeling the elements of the set $2 \cdot \omega$, we obtain ω. Hence, $2 \cdot \omega$ is again simply ω itself. In contrast, $\omega \cdot 2$ is clearly different from ω (i.e., $\omega \cdot 2 \neq \omega$), because $\omega + 3$, for example, is an element of the set $\omega \cdot 2$, but is clearly not an element of the set ω. Therefore, we can conclude that $2 \cdot \omega \neq \omega \cdot 2$ and, consequently, that multiplication of ordinals is not commutative. Visual descriptions of $\omega \cdot 2$ and $2 \cdot \omega$ can be found in Figures 11 and 12, respectively.

For more details on the formal study of ordinal numbers and ordinal arithmetic, I refer the interested reader to Chapter 6 of Hrbacek & Jech's book *Introduction to Set Theory [51]*.

6 Ordinal Numbers as a Case Study in Visualization of Mathematics

As demonstrated in Sections 1, 3, and 4, visualization plays an important role within different aspects of mathematical practice, especially when it comes to developing intuition, gaining new knowledge and understanding mathematics. Over the last few decades, growing attention among mathematicians and philosophers of mathematics has been given to bridging the gap between philosophy of mathematics and mathematical practice [8]. Alongside a shift in the approach towards the role that visual thinking and visualization play within mathematical practice, a renewed interest in framing this role, as well as its advantages and limitations, has consequently arisen [4]. However, the research on the matter is still relatively narrow, and stems mostly from a theoretical, philosophical standpoint. Furthermore, empirical studies to support or refute those theories are even more scarce [6, 7]. Therefore, for the purposes of this paper, a qualitative and quantitative study around ordinal numbers and related mathematical concepts was conducted among university and graduate students. The aim was to test the importance of visuals within mathematical practice, specifically their significance in understanding mathematics and gaining new knowledge.

By using ordinal numbers as a demonstrative case study, the questionnaire and the accompanying test attempt to verify whether integration of visuals contributes to the understanding of mathematical concepts and, if so, to what extent. The topic of ordinals was chosen due to its abstract theoretic nature; it is a challenging topic that some may find difficult to grasp as it has no vivid evidence in nature that we can see. Moreover, at the time of publication of this paper, no study that discusses visualization of ordinals, theoretical nor empirical, was found (with the exception of one paper [16], which was not empirical). Thus, successfully implementing visualization tools to support reasoning, justification for truths and understanding of concepts relating to ordinals is very relevant not only for the purposes of this paper, but also to cultivate the academic discussion around the role of visualization of mathematics on a larger scale.

6.1 Study Goal, Methodology and Set-Up

The main goal of the survey was to assess the effect of integrating visual aids when introducing new mathematical concepts on the comprehension level of participants. This was done under the hypothesis that visualization will enhance participants' understanding, as was demonstrated in previous studies (e.g., [1, 37, 38]). Hence, when introduced to the topic of ordinals, we expected that participants who encountered visual support would exhibit better test results compared to participants who did not.

The survey was divided into four parts (as detailed below) that consisted of both closed and open-ended questions. This was done intentionally, to evaluate not only the practical, direct effects of visuals on participants' performances, but also the way in which it affected participants' perceptions and attitudes towards learning mathematics. This also allowed for both qualitative and quantitative analysis of the findings.

The survey was divided into four sections:
1. Background questions
2. Information segment about ordinal numbers
3. Test: comprehension questions
4. Self-reflection questions

6.1.1 Background Questions

The first part of the survey encompassed a set of questions regarding participants' backgrounds. This included questions about their age, gender, former education, comfortability level when it comes to mathematics, and familiarity with ordinal numbers[7]. This set of questions aimed to screen out people who did not meet the requirements for taking part in the survey. Moreover, at a later stage, the data collected about participants' backgrounds was used for a more advanced and interesting analysis (e.g., comparing the different effects of integrating visuals on the performances of the various population groups).

6.1.2 Information Segment

This section included three informative segments that introduced the topic of ordinal numbers:

1. The *nonvisual* segment, which contained only textual explanations.

2. The *visual* segment, which contained mainly graphic explanations (i.e., figures and images).

3. The *mix* segment, which incorporated both the visual and nonvisual segments and contained both textual and graphical explanations.

Every participant was randomly assigned to one of the segments (making sure that each segment received an approximately equal number of participants). They were then asked to read the information carefully and answer a set of questions based on the information they received. The three segments were designed to directly test the study's hypothesis and to assess the utility of visualizations of mathematics. The first two segments (nonvisual and visual) represented the two end cases (i.e., no imagery at all vs primarily imagery), while the

[7] The answers recorded for the last question were later disregarded, as the results showed no significant difference in test performances between participants who stated to know ordinal numbers to those who do not.

third segment (mix) served as the 'control'. This allowed for not only a binary assessment of whether or not visualization contributes to participants' comprehension of the subject, but also an assessment of the magnitude of its effects. The concepts presented to participants correspond to the information provided in section 5, and as such, included concepts such as the successor operation and ordinal arithmetic. Figures 13, 14 and 15 provide an example of how the concept of infinite ordinals was presented across the three segments.

The survey was intended for people from different mathematical backgrounds, and therefore some of the formal definitions were adjusted and/or simplified to accommodate all participants.

The set of questions was identical across all three groups. Participants had the option to go back and revise the information segment as they pleased. This part of the survey aimed to test the effects of integrating visuals on participants' understanding when encountering new material in mathematics. Overall, the nonvisual group included 21 participants, the mix group included 25 participants and the visual group included 20 participants.[8]

6.1.3 Test

All participants were presented with a set of 13 questions[9] that were based on the information provided to them in the previous section. The questions were the same for all participants, regardless of their group. The questions dealt mainly with ordinal arithmetic, non-commutativity of ordinal addition and multiplication, and set-theoretic representations of ordinals. For multiple choice questions, the order of appearance of the possible answers was randomized to minimize bias. All questions are presented in the table in Figure 16. This section was intended to test participants' level of understanding of the material, and to assess and compare their performance, based on their assigned informative segment and overall success rate in the test. In accordance with the study hypothesis, it was expected that the group who received the visual information segment would overall record the highest scores in the test question section.

6.1.4 Self reflection

This section included a set of self-reflection questions in which participants were asked to assess their level of understanding of the topic and success rate in the test, report which concepts they found to be most difficult and describe their study habits with respect to the use of visualization in their studies. In addition, each participant was asked to share their opinion about the role of visualization within mathematics and to evaluate the contribution of the textual and/or visual parts to their understanding (depending on the type of informative segment they received). For instance, the mix group was asked whether they found the visuals or textual explanations more useful. Meanwhile, the nonvisual group was asked whether they tried to visualize any of the concepts themselves and if they thought that adding visuals would improve their understanding. Finally, the visual group was asked if they thought that adding textual explanations would improve their understanding. All questions can be found in Figure 17. The purpose of this section was to add another level of complexity to the analysis, as well as to gain insights into how different people perceive the role of visualization, especially in terms of gaining knowledge and understanding mathematics, and whether their perception correlates with their test performance.

6.2 Results

The survey was conducted online during the month of April 2022 among 66 university and graduate students from different cultural and educational backgrounds, all between the ages of 18-30. Overall, 38 men and 27 women participated in the study. One participant chose not to disclose their gender. All participants were required to have completed at least high school level mathematics, ensuring that all participants had a foundational understanding of mathematics, specifically arithmetic.

The table in Figure 18 presents the overall success rate of participants for each question in the test section in total and by group, according to the information segment they received. Each question's number corresponds to the question number that appears on the list of test questions in Figure 16. The 'Right' column presents the number of participants who answered the question correctly, and the 'Wrong' column presents the number of participants who answered the question incorrectly. 'Prop Right' presents a fraction between 0 and 1 which corresponds to the number of correct answers divided by the number of all given answers (i.e., their proportion). Contrary to expectations, according to hypothesis testing, there appears to be no significant difference in performance across the three groups. On average, the total proportion of correctly answered questions for each of the three was 74%, as can be seen in the 'TOTAL' row in Figure 18.

Figure 19 shows the distribution of participants according to their test scores. On average, participants answered approximately 9 out of 13 questions correctly[10]. Figure 20 presents the success rate distribution of participants according to their group and the total number

[8] The slight difference in the number of participants between the three groups stems from incomplete survey responses that were screened out during the analysis stage.

[9] Overall, there were 13 possible questions, but question 7 only appeared on the test for participants who answered question 6 correctly; therefore, some participants were asked to answer only 12 questions.

[10] The exact average value was 9.303.

Ordinals allow us to continue the process of 'counting' beyond the natural numbers; i.e., they allow us to describe and compare positions of infinite entities. One can guess that this property suggests that there is no singular infinity, rather there are different sizes of infinities, such that we can count and compare them to one another.

Infinite ordinals

The first infinite ordinal (i.e., the first number that comes **after** all natural numbers) is *omega*, denoted by the Greek letter **ω**, which is defined as the set (a collection of elements) **ℕ** of all natural numbers; i.e., ω = {0, 1, 2, 3, ...}

Figure 13: Introducing infinite ordinals in the survey's nonvisual segment

Infinite ordinals

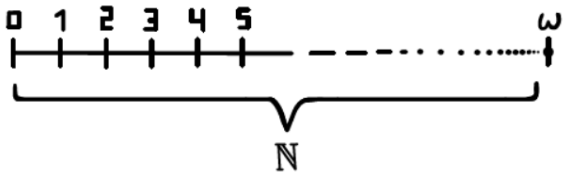


Image 2: *Omega* ω is the first infinite ordinal number that comes **after** all the natural numbers

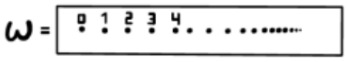


Image 3: ω is the set (i.e., a collection of elements) containing all natural numbers. i.e., ω = {0, 1, 2, 3, ...}

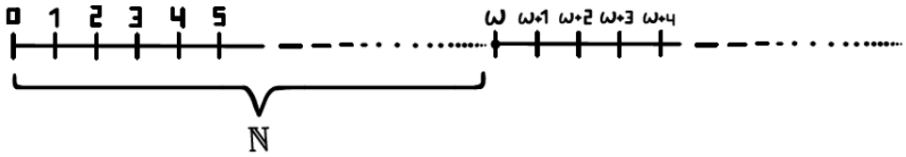
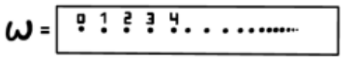


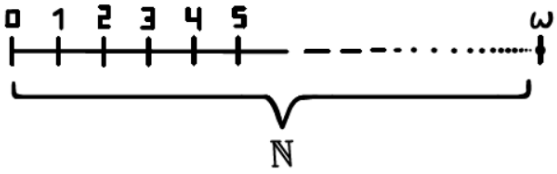
Image 4: Ordinals allow us to continue counting and comparing positions of infinite elements that are beyond the natural numbers

Figure 14: Introducing infinite ordinals in the survey's visual segment

Infinite ordinals

The first infinite ordinal (i.e., the first number that comes **after** all natural numbers) is *omega*, denoted by the Greek letter **ω**, which is defined as the set (a collection of elements) **ℕ** of all natural numbers; i.e., ω = {0, 1, 2, 3, ...}





Ordinals allow us to continue the process of 'counting' beyond the natural numbers; i.e., they allow us to describe and compare positions of infinite entities. One can guess that this property suggests that there is no singular infinity, rather there are different sizes of infinities, such that we can count and compare them to one another.

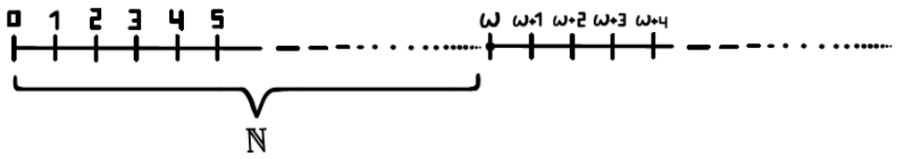


Figure 15: Introducing infinite ordinals in the survey's mix segment

Question Number	Test Question
1	True or false: $\omega + 1 = \omega + 2$
2	True or false: $412 + \omega = \omega$
3	True or false: For an ordinal number $\alpha < \omega$ (i.e., a finite ordinal), $5 * \alpha = \alpha * 5$
4	True or false: $5 * \omega = \omega * 5$
5	True or false: $4 * \omega = (1 + 1 + 1 + 1) * \omega = 1 * \omega + 1 * \omega + 1 * \omega + 1 * \omega = \omega + \omega + \omega + \omega$
6	True or false: Every ordinal number has a predecessor
7	If you answered 'No' in the previous question, please provide an example of such ordinal
8	Please type your answer in the box below: Express the number 5 in terms of successors of 0 [Hint: $8 = S(S(6))$]
9	Multiple choice: For an arbitrary ordinal number α, which of the following is a representation of $\alpha * 4$?
10	Multiple choice: Which of the following is a representation of $\omega * 3$ in set notation (Hint: recall that $\omega = \{0, 1, 2, 3, \ldots\}$)?
11	Multiple choice: For an arbitrary ordinal number α and a natural number $n \in \mathbb{N}$, which of the following best represents the ordinal number $\alpha * n$?
12	Multiple choice: For any pair of ordinal numbers α, β, which of the following could never be an element in the set representation of the ordinal number $\beta * \alpha$?
13	Multiple choice: Which of the following is an element in the set representation of the ordinal number $\omega * 4$?

Figure 16: Test Questions

Group	Self-reflection Question	Answer Type
Visual	To what extend did you find the visuals (i.e., the images and diagrams) helpful?	Extremely useless – Extremely useful scale
Visual	Do you think that integrating more textual descriptions in the explanation section would improve your understanding of the topic and consequently your performances?	Definitely not – Definitely yes scale
Mix	To what extend did you find the visual parts (i.e., the images and diagrams) of the explanation helpful?	Extremely useless – Extremely useful scale
Mix	To what extend did you find the verbal parts of the explanation helpful?	Extremely useless – Extremely useful scale
Mix	Which part did you find to be more useful?	Multiple choice
Nonvisual	Do you think that integrating visuals in the explanation section would improve your understanding of the topic and consequently your performances?	Definitely not – Definitely yes scale
Nonvisual	Have you tried to visualize some of the definitions/questions, by sketching a diagram for example?	Yes – No
Nonvisual	If you answered 'Yes' in the previous questions, please elaborate below:	Open question
All groups	To what extent do you agree with the following statement: I feel like I understood the topic and was able to answer the questions properly	Strongly disagree – Strongly agree scale
All groups	How well do you think you performed?	Extremely bad – Extremely good scale
All groups	Which question type did you find to be most challenging? You can select more than one option	Multiple choice
All groups	Do you often use visualization to clarify, explain and understand learning material in your studies or work, especially with regard to mathematical material?	Never – Always scale
All groups	Do you think visual representation can (and should) play a role in learning mathematics?	Definitely not – Definitely yes scale
All groups	Please elaborate your previous answer: How do you see the role of visualization in mathematics?	Open question

Figure 17: Self-reflection questions per group

of correct answers. On average, the mix group obtained the highest score (9.4), but not by a significant difference; the visual and nonvisual groups obtained average scores of 9.25 and 9.23, respectively.

Figures 21 and 22 present participants' distribution according to their area of study. As can be seen in the figures, most participants (42 overall) had a background in mathematics, 17 participants had a background in other scientific fields and only 7 participants reported having a background in social science (5) or humanities (2).

Contrary to expectations, the test results showed no significant difference across the three groups. However, when participants were asked to report how well they thought they understood the topic (as part of the self-reflection section), the nonvisual group appeared to be the least confident regarding their understanding. Figure 23 shows that 71.4% of participants in the nonvisual group agreed with the statement *I feel like I understood the topic and was able to answer the questions properly to some extent*, in comparison to 75% and 84% in the visual and mix groups, respectively. The pie charts in Figure 24 present the response distribution of participants across the three groups for the self-reflection question *Which question type did you find to be most challenging?*

As can be seen in Figure 25, all participants agreed to some extent that visualization plays an important role in learning mathematics. Not surprisingly, participants who were part of the visual group were most certain regarding its role, where 75% agreed unequivocally with the statement *visual representation can (and should) play a role in learning mathematics*; compared to only 56% and 52.4% in the mix and nonvisual groups, respectively.

The table in Figure 26 presents the average answer of each group for each of the four self-reflection questions which were presented to all participants. The numerical values appearing in the table correspond to the following scale:

For the first two questions, on a scale of 1-7, the corresponding values are as follows:

1 - Strongly disagree / Extremely bad
2 - Disagree / Moderately bad
3 - Somewhat disagree / Slightly bad
4 - Neither agree nor disagree / Neither bad nor good
5 - Somewhat agree / Slightly good
6 - Agree / Moderately good
7 - Strongly agree / Extremely good

For the last two questions, on a scale of 1-5, the corresponding values are as follows:

1 - Never / Definitely not
2 - Sometimes / Probably not
3 - About half the time / Might or might not
4 - Most of the time / Probably yes
5 - Always / Definitely yes

It is apparent from Figure 26 that, on average, participants who were part of the visual group were most comfortable with the subject (regardless of their test performance). They also expressed the most positive attitude towards the role of visualization in mathematical education. By contrast, participants who were part of the nonvisual group showed the least confidence in their test performance and understanding of the subject, as well as in the role of visualization in learning mathematics. This confirms the hypothesis that increasing the integration of visual aids when introducing a new mathematical topic enhances participants' confidence and positively affects their attitude towards learning mathematics.

Together these results provide important insights into the utility of visualization of mathematics. Contrary to expectations, the results do not indicate a clear connection between participants' level of exposure to visual means and their success rate in the test. This could be due to a number of limitations identified in the study, which are further discussed in Section 6.5. Nonetheless, the results suggest that there exists a positive correlation between the presence of visuals in introductory segments and participants' confidence levels, as well as between the former and participants' perspective on visualization of mathematics. These observations and their implications are further discussed in the following section.

6.3 Analysis and Discussion

Overall, the results did not provide concrete evidence for a significant difference in performance across the three groups. However, it is important to note that apart from the two open-ended questions, all questions displayed in the test part of the survey had a closed set of possible answers. Therefore, even if participants did not know the correct answer, they had a 25% or 50% chance of guessing correctly (depending on the number of possible answers). This detail, alongside the relatively small sample size of each group, implies that the test results alone do not necessarily accurately reflect the effect of integrating visual aids on participants' understanding of the subject matter. This conjecture is further reinforced when comparing the overall performance of participants who reported that they are familiar with ordinal numbers and participants who reported the opposite. Despite the reasonable assumption that the former group would perform better than the latter on the test, regardless of their assigned information segment, statistical testing indicates that this variation in performance is not as significant as expected. While the group of participants who indicated that they are not familiar with ordinals obtained, on average, 72.1% of correct answers, the group of participants who indicated that they are familiar with ordinal numbers performed only slightly better, with an average success rate of 74.79%. This observation raises the possibility that participants interpreted the notion of familiarity as something they have heard

Question number	Total (66 participants)			Visual (20 participants)			Mix (25 participants)			Nonvisual (21 participants)		
	Right	Wrong	Prop Right	Right	Wrong	Prop Right	Right	Wrong	Prop Right	Right	Wrong	Prop Right
1	59	7	0.89	19	1	0.95	21	4	0.84	19	2	0.90
2	51	15	0.77	15	5	0.75	21	4	0.84	15	6	0.71
3	60	6	0.91	17	3	0.85	24	1	0.96	19	2	0.90
4	56	10	0.85	18	2	0.90	23	2	0.92	15	6	0.71
5	43	23	0.65	14	6	0.70	17	8	0.68	12	9	0.57
6	42	24	0.64	11	9	0.55	15	10	0.60	16	5	0.76
7	37	5	0.88	10	1	0.91	14	1	0.93	13	3	0.81
8	55	11	0.83	19	1	0.95	19	6	0.76	17	4	0.81
9	58	8	0.88	17	3	0.85	24	1	0.96	17	4	0.81
10	39	27	0.59	12	8	0.60	12	13	0.48	15	6	0.71
11	22	44	0.33	7	13	0.35	5	20	0.20	10	11	0.48
12	46	20	0.70	13	7	0.65	20	5	0.80	13	7	0.65
13	46	20	0.70	13	7	0.65	20	5	0.80	13	7	0.65
TOTAL:	614	220	0.74	185	66	0.74	235	80	0.75	194	72	0.73

Figure 18: Survey results - overall success rate per test question

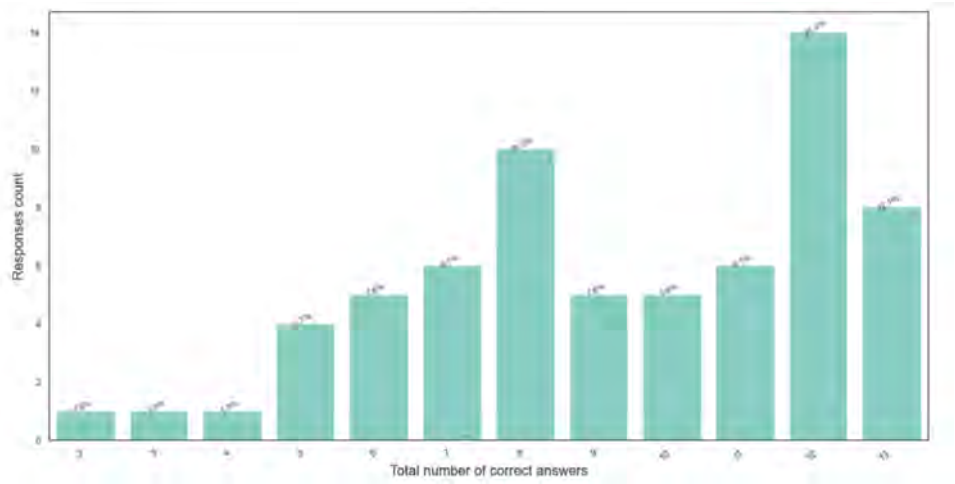


Figure 19: Distribution of participants' test scores (where the test scores range from 0 to 13)

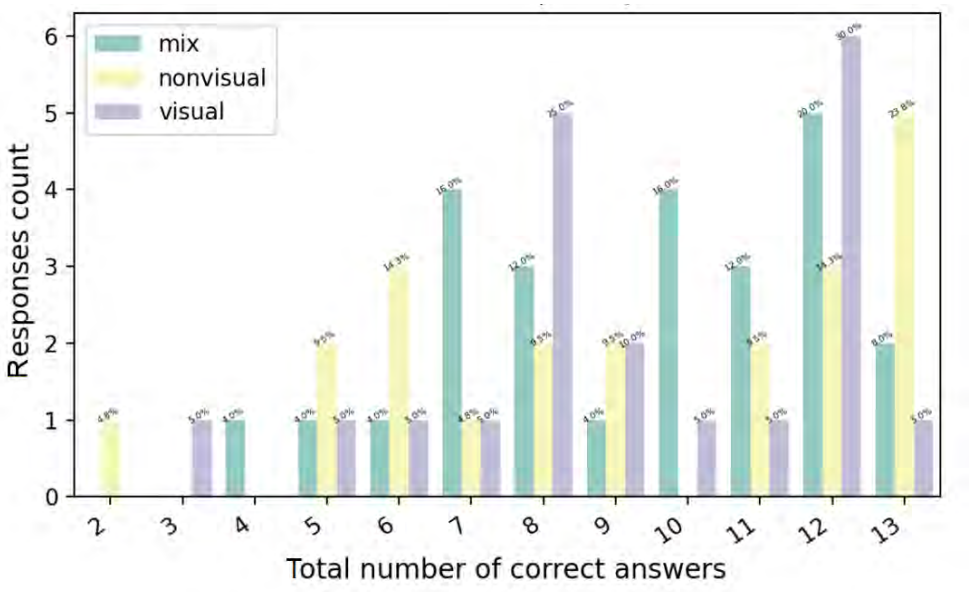


Figure 20: Participants' test scores distribution by groups

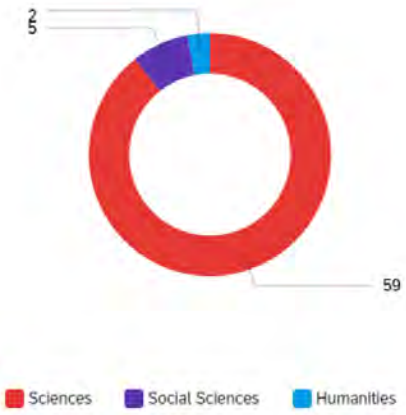


Figure 21: Pie chart distribution of participants' area of study

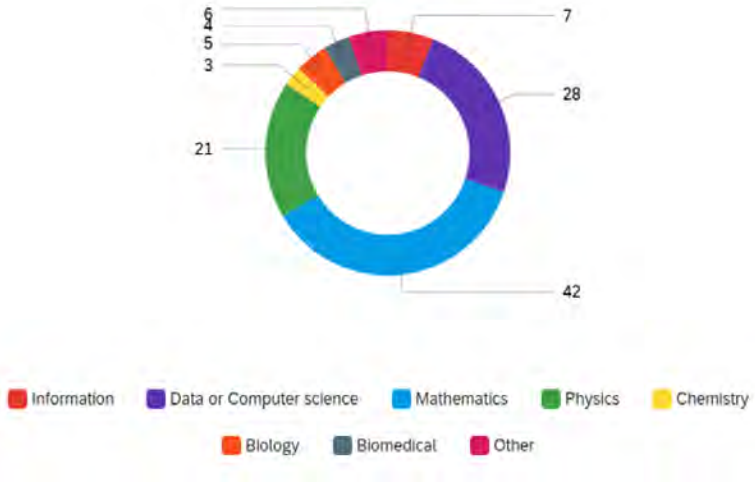


Figure 22: Pie chart distribution of participants' area of study for those who have a scientific background.
Note: Participants were allowed to choose more than one area of study and therefore the overall sum of this pie chart exceeds the total 66 participants.

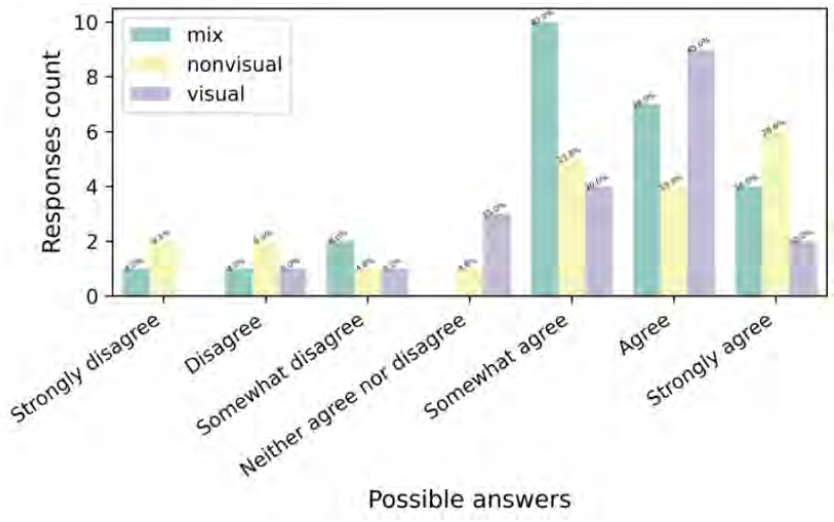


Figure 23: Distribution of participants' response to the self-reflection question *To what extent do you agree with the following statement: I feel like I understood the topic and was able to answer the questions properly*

Figure 24: Pie charts distribution of participant's answers to the self-reflection question *Which question type did you find to be most challenging?* with respect to their group. From top to bottom: visual group, mix group, nonvisual group

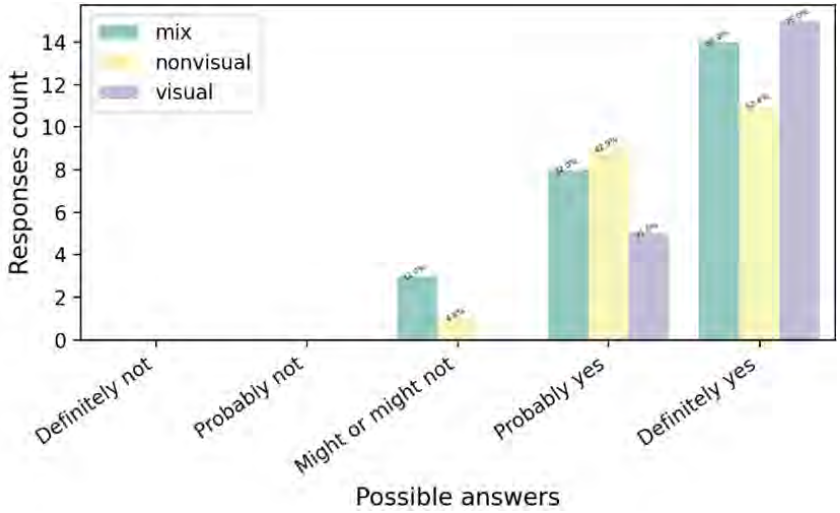


Figure 25: Distribution of participants' response to the self-reflection question: *Do you think visual representation can (and should) play a role in learning mathematics?*

Self-reflection Question	VISUAL	MIX	NONVISUAL	TOTAL
To what extent do you agree with the following statement: I feel like I understood the topic and was able to answer the questions properly	5.25	5.16	4.95	5.12
How well do you think you performed?	4.9	4.92	4.63	4.82
Do you often use visualization to clarify, explain and understand learning material in your studies or work, especially with regard to mathematical material?	3.9	2.96	3.24	3.33
Do you think visual representation can (and should) play a role in learning mathematics?	4.75	4.44	4.48	4.55

Figure 26: Self-reflection average answer per group

about before rather than a more thorough acquaintance with the subject and its related concepts. Therefore, the answers recorded for this question were not considered in the analysis.

Another feasible explanation for the lack of significant difference in success rate between the three groups is the effect of participants' mathematical background on their performance. When comparing the overall performance of participants who reported their main area of occupation to be mathematics to those who came from other academic backgrounds, there is a significant difference between the two. On average, participants with mathematical background (42 participants) answered 78.76% of the test questions correctly, compared to only 64.57% of correct answers for participants with different backgrounds (24 participants). This suggests that prior mathematical knowledge has a greater effect on performance than the type of information segment participants received, especially when the sample sizes for each group are relatively small. Furthermore, when comparing the effect of the different information segments only on the performance of participants without a strong mathematical background, there seems to be a more significant difference between the three groups, especially between the mix and nonvisual groups. However, this sample is too small (i.e., 8 participants on average for each information segment) to draw unambiguous conclusions. Similar findings were observed when conducting a comparison between the performances of male and female participants; on average, the male group answered 77% of the test questions correctly, while the success rate of the female group was 63%. Nonetheless, this variance is most likely stemmed from participants' mathematical background rather than their sex; out of 27 female participants, only 12 obtained a background in mathematics (i.e., 44.4%), compared to the men's group, where 29 out of 38 participants had a mathematical background (i.e., 76.3%).

The test results indicate that participants across all three groups struggled the most with questions involving set-theoretic membership and representation of ordinals. This observation is most evident in the low success rate obtained in question 11 in the test section (6.1.3)[11], where only 33% of all participants answered this question correctly. Given that set theory is an abstract field in mathematics that is often considered difficult to grasp, it is plausible to assume that the visual aids used to illustrate set-theoretic membership and representation of ordinals in the visual and mix information segments were insufficient, and that employing more sophisticated illustrations could increase participants' comprehension.

Interestingly, the results also suggest that participants who were in the mix and visual groups understood the rules of commutativity of ordinals better than the nonvisual group. This can be seen in questions 4 and 5 in the test section (6.1.3)[12], where the success rates of the visual and mix groups in question 4 were 90% and 92% respectively, while only 71% of participants of the nonvisual group answered this question correctly. Similarly, only 57% of participants of the nonvisual group answered question 5 correctly, compared to 70% and 68% of participants of the visual and mix groups, respectively. This outcome may suggest that the visual aids used to demonstrate commutativity rules of ordinals were efficient and provided the relevant participants an intuitive sense of the topic.

These findings are well-aligned with the responses recorded for the question *Which question type did you find to be most challenging?*, which appeared in the self-reflection section (6.1.4): as anticipated, all three

[11] Multiple choice: For an arbitrary ordinal number α and a natural number $n \in N$, which of the following best represents the ordinal number $\alpha \cdot n$?

[12] Question 4: True or false: $5 \cdot \omega = \omega \cdot 5$?
Question 5: True or false: $4 \cdot \omega = (1 + 1 + 1 + 1) \cdot \omega = 1 \cdot \omega + 1 \cdot \omega + 1 \cdot \omega + 1 \cdot \omega = \omega + \omega + \omega + \omega$?

groups found the set-theoretic membership type of questions to be most challenging (mix – 29.73%, visual – 29.17%, nonvisual – 23.08%). Furthermore, the same proportion of participants of the nonvisual group (i.e., 23.08%) reported that they found the ordinal multiplication type of questions to be most challenging (for comparison, only 12.5% of the visual group and 18.92% of the mix group reported this question type to be most challenging). This corresponds to their relatively low success rate on questions 4 and 5 in the test.

Moreover, the highest percentage of participants who claimed to find all types of questions difficult was observed in the nonvisual group (19.23%, in comparison to 12.5% and 8.11% in the visual and mix groups, respectively). On the contrary, the highest percentage of participants who claimed they did not find any type of question difficult was in the visual group (16.67%). For a complete picture of the response distribution for this question across the three groups, see Figure 24 in Section 6.2.

Even though statistical hypothesis testing showed no significant difference in the overall performance, this may suggest that participants who had the visual information segment felt more comfortable with the material compared to those who had only textual explanations. It is possible that for a similar, larger-scale study, comfortability levels would translate into better scores among participants of the visual group (and, respectively, lower scores among participants of the nonvisual group).

6.4 Qualitative Discussion

As part of the self-reflection section of the survey, participants were asked to answer the open-ended question *How do you see the role of visualization in mathematics?* Interestingly, the answers recorded from this question are consistent with the academic discourse presented in Section 3; while a majority of participants reported a positive attitude towards visualization of mathematics and acknowledged its important, legitimate role, several respondents presented a more careful, skeptical approach regarding the magnitude of its effect.

Many participants mentioned that visualization, and visual thinking in general, play an important role when it comes to intuition and gaining knowledge. As one participant put it: *"if you can create an accurate visual stimulus in your mind, the understanding must fall into place quickly"*. Other participants shared similar observations, stating that visualization is *"of an extreme importance as it gives the intuition behind the concept"*, and *"one can understand a mathematical idea only after one has visualized it"*.

Nevertheless, some mentioned that while they see visualization as an indisputably important tool that enhances understanding and intuition, it is not always necessary and often not rigorous enough on its own; *"a visualization speaks more than a sea of symbols, although non-rigorously"*, it is a *"complimentary explanation"*, that *"provides the ability to prove informally..."*.

Other respondents emphasized the importance of visualization when learning mathematics, stating that *"it should be included in the school system"* and stressing how integrating more visual explanations in mathematics studies can *"help students feel more comfortable, less threatened by the new subject and develop a mathematical sense"*. Another participant postulated that *"if we'll use more visual aids in teaching and learning mathematics, we can help people study it faster, more efficiently, and even to attract more people into the mathematics and various sciences fields"*. Intriguingly, this view is congruous with the findings presented above (6.2), strengthening the hypothesis that visualization can increase one's motivation to learn, boost one's confidence, and enhance understanding, which, in turn, can help students overcome math phobia. The fact that all statements presented in this section were written by students and alumni reinforces the scarcity of visualization of mathematics in higher mathematics education and its unrealized potential. One participant took this idea one step further, concluding *"...that (at the very least) every new term or definition taught should be explained with a visual example"*. Another participant proposed a slightly different approach; while they agreed that visualization plays an important role *"as an integral part of the learning process"*, they also argued that *"it is extremely important for the student to come up with his own, original visual representation of the material"*.

Several participants mentioned that they believe the role of visualization varies based on either the person (*"...some people are more visual, others auditive..."*) or the mathematical field. A few respondents mentioned that, in set theory specifically, they do not think it plays such an important role (as opposed to calculus, algebra, geometry, and topology). It is interesting to note that, in this context, one participant mentioned they are *"...not sure about visualization in set theory, but it might be because I've never seen one [sic!]"*. Unsurprisingly, this participant was part of the nonvisual group. On the contrary, a participant from the visual group claimed the opposite; *"...In the latter [set-theory], visual aid helps clarify the new definitions and the innate contradiction we have"*. These two responses not only further indicate *"that visual learning is so underrepresented in our learning of mathematics"*, as manifested in the former, but also suggests that exposing students to such tools (as done in this survey) may affect their perception of visualization in mathematics and, consequently, diversify (and improve) their study methods, especially when developing intuitive thinking and gaining new knowledge.

These results provide further support for the hypothesis that visualization is a useful, essential tool in mathematical practice, presumably beyond its clearly evident heuristic role. Notwithstanding, as manifested in participants' responses and the inconclusive test results, the

ideal method of integrating visual aids in mathematical studies, or in other realms of mathematical practice, is still unclear. Furthermore, it could conceivably be hypothesized that different purposes (i.e., discovery, justification, understanding, etc.) will require different methods and types of visualization. Further research should be undertaken to investigate different types of visualization of mathematics and their advantages and limitations within the previously mentioned aspects of mathematical practice.

Nevertheless, the fact that the results of the test section of the survey did not provide a clear conclusion does not mean we cannot draw conclusions about the impact visualization has on the way participants perceived the studied material and the learning experience itself.

Mathematics is a marathon, not a sprint. At the very best, visualization can improve understanding, used as a means of discovery and some might even say as means of justification; at worst, it may not have any meaningful effect on performance or the path one should take to reach the solution, but at least it will make the journey more attractive and inviting. As delightfully concluded by one of the participants: *"Visualization, I believe, gives more permanent understanding; A function behaves in a certain way...and visualizations represent their actions. Otherwise, math learning is just computation"*.

6.5 Limitations and Recommendations

The findings of this study need to be considered in light of some limitations. First, the research involved a relatively small sample group (66 participants), which made it difficult to determine the significance of the conclusions. This was especially problematic when dividing responses into sub-groups (i.e., according to their area of expertise) to disclose differences in their performance across the three information segments. These divisions yielded very small sample sizes, where clear conclusions could not be drawn as outliers could greatly impair the results. Future studies should strive for a bigger data set to facilitate such divisions and allow more advanced analysis, which could lead to novel, more reliable insights.

Second, the informative segments presented in the survey were designed to accommodate participants with different backgrounds in mathematics; it is possible that trying to tailor the segments for different levels of mathematical knowledge harmed their clarity and caused participants to suffer from information overload (or underload). Due to the nature of this survey, it mainly attracted participants with a mathematical background, where more than half of the participants (56%) reported that they were familiar with ordinal numbers prior to participating in the survey. This might harm the reliability of the results, as it is possible that prior knowledge had a greater effect on participants' performance rather than the information segment type they received. Future research should consider excluding such participants to avoid bias. Alternatively, targeting a more homogeneous group, where participants share a similar mathematical background, and tailoring to-the-point explanations that match their mathematical knowledge, could minimize bias and participants' presumptions about the topic.

Third, as the survey took place online, controlling the research setting was unfeasible and, consequently, there was no supervision of participants while filling out the survey. Thus, it was not possible to distinguish between random guessing and false understanding, or to determine whether external sources were in use. Similar future research should consider conducting a supervised study, where participants answer the survey within a specified time frame, in a designated location. Alternatively, the option "I don't know" could be added for multiple choice questions, to eliminate guessing.

Lastly, the focus of the survey was the effects of integrating visual aids on participants' understanding of ordinal numbers, a specific niche of mathematics; making it hard to generalize the findings and their implications for other areas of mathematics. Therefore, more rigorous, large-scale research should take place to verify (or refute) the results obtained in this study and to expand its scope to other areas of mathematics. This could provide new insights and further promote the academic discussion concerning the epistemic role of visualization within mathematical practice and formal mathematics.

7 Conclusion

The main aim of this research was to assess the significance of visualization of mathematics within different aspects of mathematical practice, especially within the theme of ordinal numbers. A secondary objective was to examine the different contemporary views and attitudes towards the role visualization could and should play within the realm of mathematics. As demonstrated throughout this paper by various examples, visuals are an integral part of mathematics, from discovery and gaining knowledge to justification and understanding. This study has identified a welcoming change in the approach towards visualization of mathematics among scholars, educators, and students of mathematics via both theoretical (Section 3) and empirical (Section 6) discussions. Nevertheless, the findings signify that the scope of the epistemic role of visualization is still a subject for debate that is yet to receive the attention it deserves in academia.

Indeed, this study appears to be the first to investigate the impact of visuals on students' comprehension of ordinal numbers empirically. Whilst the empirical study did not unequivocally confirm the beneficial effects of integrating visuals when introducing a new mathematical topic on participants' understanding, it did substantiate its effects on participants' percep-

tion. The qualitative analysis has provided new insights into how visualization of mathematics affects the way in which students perceive its role, as well as their understanding of the studied material. Participants who were not exposed to visual aids tended to underestimate their level of understanding and success on the test. This observation raises several interesting suppositions: could integrating visualization in mathematical education increase students' confidence levels, help students and scholars develop an intuitive sense, combat math phobia, and even attract young adults to the field?

In light of the evidence presented, the author advocates increasing the integration of visualization within various domains of mathematical practice, especially in education, as it seems to be beneficial in developing a positive attitude towards mathematics at the very least. However, this was a relatively small-scale study that focused on a very specific sub-field of mathematics. With the fast-paced advancements of technology, and especially computer-generated graphics, it is most likely that the significance and potential benefits of visualization in mathematics will only increase with time. Therefore, further research is vital in order to frame the epistemic role of visualization and deduce unambiguous conclusions about its significance. Analyzing, framing and defining a clear set of guidelines, along with formally and systematically clarifying the advantages and limitations of using visualization within different realms of mathematics would greatly benefit the mathematical community and beyond.

Acknowledgmets

This paper and the research behind it would not have been possible without the exceptional support and encouragement of my supervisor, Yurii Khomskii. I am extremely grateful for your endless patience and support of my sometimes naively ambitious ideas. Thank you for your feedback and guidance in every stage of the research and for sharing your valuable and constructive insights with me, which greatly contributed to the final product. I would also like to show my gratitude to the 66 anonymous participants for taking the time to answer the survey. Lastly, I would like to thank my parents and siblings for their unwavering support and my friends and family for putting up with my endless talks about the subject matter (sometimes against their will) and still sticking around.

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Mobile Money in Afghanistan - Potential and Limitations for Transformation

Analyzing the robustness of the M-Pesa framework through the case study of Afghanistan's mobile money system M-Paisa

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Abstract

Mobile money has been a key component of the globally increasing rate of financial inclusion. The digital FinTech service can provide the possibility to offer peer-to-peer transactions at the most basic level up to creating savings, taking out micro-loans, and purchasing insurance products. By relying solely on the mobile phone to access the service, physical obstacles to financial inclusion are circumvented. The mobile money service M-Pesa in Kenya was highly successful, resulting in poverty reduction and greater financial stability for users. M-Paisa, the mobile money service in Afghanistan, was launched shortly after but did not reach M-Pesa's scale. Research has omitted in-depth investigation as to why the service did not reach the assumed scale and if the framework provided by M-Pesa or other factors influenced the expansion of M-Paisa. This study aimed to answer the research question: Has the framework of the mobile money system M-Paisa in Afghanistan been robust enough to effectively foster local financial inclusion? By combining a literature review with expert interviews, the study was able to deduce that the framework provided by M-Pesa cannot be held responsible for M-Paisa's outcomes. Rather, the analysis suggests that country-specific factors of Afghanistan such as the low technological development and infrastructure, lack of trust in financial institutions, socioeconomic factors, a constantly high level of violence, lack of scale, and competition with the Hawala network were countervailing exogenous factors. Since Afghanistan is a country of extreme socioeconomic levels and high levels of violence, the findings from this study are not generalizable to the efficacy of mobile money as a development tool, but highlight the importance of taking country-specific circumstances into account when launching such services.

Keywords and phrases: *mobile money, M-Paisa, Afghanistan, financial inclusion, M-Pesa, FinTech*

Suggested citation:
Ziener, L.M. (2022). Mobile Money in Afghanistan - Potential and Limitations for Transformation: Analyzing the robustness of the M-Pesa framework through the case study of Afghanistans's mobile money system M-Paisa. *AUC Undergraduate Journal of Liberal Arts and Sciences*, 19(1) pp. 63-85

imprint.

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1 Introduction

The rapid growth of digital technologies in the past decades has offered novel chances and opportunities in many areas such as development, finance, and communication. Digital financial technologies known as FinTech have emerged as a central driving force in disrupting traditional financial transactions (Demir et al., 2020; Makina, 2019). For the fulfillment of 10 out of the 17 Sustainable Development Goals (SDGs) by 2030, which are aimed at combating poverty and climate change while improving global prosperity, a greater degree of financial inclusion, especially in developing countries, is essential (Arner et al., 2020). Financial inclusion entails access to financial services such as deposit or transaction accounts, credit and insurance products as well as the ability to use them to manage financial risk (Demirgüç-Kunt et al., 2018, p. 2). Mobile money, a term which refers to transactions via a mobile phone, is a tool used to foster this degree of financial inclusion globally and has been regarded as one of the most impactful drivers of financial inclusion in the last decade (Demir et al., 2020; Demirgüç-Kunt et al., 2018). The first system, called Smart Money, was introduced in the Philippines in 2001, but it was Kenya that achieved notable international success with mobile money through the system M-Pesa after its establishment in 2007 (Jack & Suri, 2014).

A study conducted by Suri and Jack (2016) found that since M-Pesa's launch, the degree of financial inclusion, especially in rural areas, has increased significantly, and poverty was reduced as a direct result of the implementation of the mobile money system. This is explained by changes in financial behavior of mobile money users through higher rates of consumption, easier access to remittances, and changes from agriculture to business professions due to higher savings and more financial mobility, especially by women, leading to higher economic shock resilience (Aron, 2018).

Simultaneously, the mobile money system M-Paisa was initiated in Afghanistan based on M-Pesa's framework and mechanisms. In light of the positive outcomes yielded by M-Pesa, development institutions were hopeful for M-Paisa to succeed (Benami & Carter, 2021; Blumenstock et al., 2015 Chipchase et al., 2011; Howell, 2012). However, its outcomes have not matched M-Pesa's in terms of key indicators of individual wealth or well-being (Blumenstock et al., 2015). The underlying reasons for these diverging results are manifold and a clear, single cause is difficult to establish. Nevertheless, when assessing multiple factors such as the historical background, level of financial literacy, and infrastructure, only a holistic analysis of the case study of Afghanistan will be able to identify key problems. This is necessary in order to understand the conditions for successful mobile payment services, especially since mobile money has been the least present in Central Asia while Africa showed the highest growth rates for mobile money accounts worldwide (Demirgüç-Kunt et al., 2018).

Presently, the majority of the academic discourse has studied and reviewed the drivers of success of mobile money systems worldwide with a focus on M-Pesa. The main emphasis in research has been on the effects of mobile money usage after experiencing shocks on a household level (Blumenstock et al., 2016; Hamdan et al., 2021). While many mobile money initiatives have been launched following M-Pesa's triumph, the outcomes have not been consistent and in-depth studies of other mobile money initiatives have been lacking (Evans & Pirchio, 2015; Herzog & Bolli, 2015). Contemporary research has not explored the individual causes for this heterogeneous development, creating a research gap in the literature. Thus, a profound understanding is needed to develop effective solutions for a higher prevalence of mobile money accounts in countries such as Afghanistan (Andersson-Manjang & Naghavi, 2021).

Hence, this study analyzes and evaluates in detail how the mobile money system M-Paisa in Afghanistan has been operating. It investigates which factors have halted greater expansion of M-Paisa using Kenya's M-Pesa system as a reference, which yielded very high rates of dispersion and adoption. Therefore, the following research question arises: *Has the framework of the mobile money system M-Paisa in Afghanistan been structurally robust enough to effectively foster local financial inclusion?* In this context, robust refers to the ability of a system, or in this case a framework, to be resilient, even when exogenous factors are impeding its functionality (O'Rourke, 2007). Considering this research question, the following sub-questions arise: If the framework is sufficiently robust, which factors outside of the framework have inhibited M-Paisa?; Are the inhibiting factors specific to Afghanistan?; Which general conclusions can be drawn from this case study?

The case study offers the possibility to analyze country-specific factors aiming to infer whether the mobile money framework, when employed in other countries with similar socio-economic factors to Afghanistan, is structurally robust and effective since research findings are scarce. The present study, which aims to provide insights into specific alterations that should be made to fit the specific socioeconomic factors of Afghanistan to enhance financial inclusion, is deduced from expert interviews and a literature review. This study intends to narrow the present research gap and highlight the interplay between mobile money systems and socioeconomic factors, particularly in regions with distinctive structural or cultural features such as Afghanistan.

Nevertheless, the case study of Afghanistan leaves limited room for generalizations, since the country has been riddled with conflicts and instability in the last four decades. These factors were taken into account during this study, and the paper aimed to identify country-

specific aspects applicable to this case study.

Consequently, the findings and conclusions from this research should support and guide decision- and policymakers in the development field when assessing the effectiveness, difficulties, and possibilities of mobile money systems, specifically M-Paisa in Afghanistan. Explanations for M-Paisa's differing results from M-Pesa should fill the gap in the academic literature that has omitted a clear analysis of Afghanistan's mobile money system. By deducing which inhibiting factors have halted M-Paisa's success and offering possible improvements, this paper will enhance the understanding of mobile money systems in areas that do not yet provide sufficient levels for mobile money systems, but showcase viable opportunities for success in the future.

2 Research Context

2.1 Financial Inclusion

Financial inclusion consists of varying degrees. Rudimentary financial inclusion entails having a deposit or transaction account at a financial institution, or mobile money platform. Such accounts are designed to enable payments and accumulate savings. Greater financial inclusion encompasses the possibility to borrow money and utilize formal insurance products. These services are particularly important when larger payments need to be made in advance for business or educational purposes. Until recently, the definition of financial inclusion did not incorporate electronic mobile money but was restricted to the formal financial sector. The inclusion of mobile money is warranted as it refers to a particularly dynamic aspect of financial inclusion (Aron, 2018; Demirgüç-Kunt et al., 2017; Demirgüç-Kunt et al., 2018).

Financial inclusion is connected to the notion that economic growth and welfare are enhanced through a higher degree of financial services being accessed, used, and incorporated in low-income countries with underdeveloped financial sectors (Khera et al., 2021). Strong positive associations have been established between higher financial inclusion and lower income inequality as well as poverty levels, substantiating the link to development benefits (Omar & Inaba, 2020).

As the Global Financial Index (Global Findex) outlined in 2017, the main advantages of financial inclusion constitute increasing income, reducing poverty, accumulating savings, and improving the management of financial risks. By relying mainly on digital technology, the cost of receiving payments is minimized by resorting to mobile phones instead of cash as the medium of transactions. This has the benefit of reducing corruption through transparent systems and offering a higher level of security to the recipient and sender (Barajas et al., 2020).

In return, this has intensified the focus of governments and international development institutions to create policies that encourage the usage of financial services of the unbanked population. In 2019, five years after the first Global Findex Report in 2014, 54 of 55 emerging markets had adopted policies that were directly aimed at improving financial inclusion, underlining the increased international awareness (Demirgüç-Kunt et al., 2018).

In addition, a gender gap in favor of males persists in account ownership. In developing countries, a constant difference of nine percent between male and female ownership has been noted. Apart from gender, global patterns also show that lower age, lower level of education, and lack of participation in the labor market are correlated with reduced account ownership (Demirgüç-Kunt et al., 2018). As a result, inclusivity within financial inclusion needs to be strengthened since the tool circumvents obstacles that previously excluded marginalized groups (Barajas et al., 2020).

In practice, financial inclusion has increased one of the most important factors: the share of the global population with a bank account. According to the Global Findex 2017, from 2011 until 2017 this share has risen from 51% to 69% worldwide. However, many of these accounts are not used for accumulating savings by households but instead for, e.g., financial transactions. On a global average in 2017, more individuals using credit cards borrowed credit in comparison to people without credit card usage. Concretely, of all people using a credit card, 23% borrowed credit compared to 22% in 2011. Of all individuals without a credit card, 11% borrowed credit compared to nine percent in 2011 (Demirgüç-Kunt et al., 2018). This does not mirror the growth in account ownership, demonstrating that basic but not extensive financial inclusion, where the latter entails active usage of the account, is expanding.

The rise in account ownership can be attributed for the most part to mobile money. It is a FinTech innovation that allows mobile phone users to set up an account at a financial institution via a mobile money platform without physically going to a bank. While mobile money providers are present on all continents, the strongest growth in mobile money account ownership has taken place in Africa, specifically in Sub-Saharan Africa with a 40% share of the total population in Gabon and over 30% in Côte d'Ivoire having an account in 2017 (Aron, 2018; Demirgüç-Kunt et al., 2018).

2.1.1 Microfinance in the Context of Financial Inclusion

Before digital technologies experienced tremendous growth and development following the 2000s, microfinance was perceived as one of the main driving forces of financial inclusion (Benami & Carter, 2021). Microfinance itself is the provision of microcredits by microfinance institutions (MFIs) dispersed to individuals with a lack of collateral. The option of taking out a loan at a formal financial institution to foster their financial sta-

bility and independence was denied to individuals with a lack of collateral (Banerjee et al., 2015; Buvininć & Furst-Nichols, 2016; Gabor & Brooks, 2017; Imai et al., 2012; Mayoux, 2000).

However, despite these promising aspects, microfinance programs yielded only mixed outcomes following their disbursement (Banerjee et al., 2015; Imai et al., 2012). Issues due to information asymmetries, a lack of credit history, and default and need-based investment of the loans impeded the large-scale positive impact of micro-credits and lowered the incentive for MFIs to distribute credit. These mixed outcomes have limited the promotion of microfinance in policymaking (Benami & Carter, 2021; Yunus, 2004). Nevertheless, through the increase in mobile phone usage and digital financial services, a new opportunity is created to relaunch microfinance via mobile phones. Therefore, the advances made by microfinance for financial inclusion and its awareness, even if lacking on a global scale, were the forerunners of the current development of financial inclusion (Benami & Carter, 2021; Buvinić & Furst-Nichols, 2016).

2.2 Mobile Money as a Tool for Financial Inclusion

Digital services and technologies created new opportunities for financial inclusion. However, on a global scale, this development did not affect everyone equally (Gabor & Brooks, 2017). While high-income countries utilized digital technologies to expand their service portfolio in existing financial institutions, a large part of the global population, especially in rural areas of low-income countries, did not have access to any form of financial services (Demirgüç-Kunt et al., 2018). In 2017, 1.7 billion people remained unbanked, and thus financially excluded. One billion of those people owned a mobile phone and 480 million had access to the Internet (Demirgüç-Kunt et al., 2018). This has provided a novel chance to reach a previously inaccessible share of the population, e.g. through mobile money (Andersson-Manjang & Naghavi, 2021; Makina, 2019). Mobile money refers to an electronic wallet system that allows financial services to be accessed only via a mobile device. These services include transfers of money (peer-to-peer), cash withdrawals, managing savings, taking out micro-loans, and purchasing insurance products (Demirgüç-Kunt et al., 2018; Donovan, 2012). To open a mobile money account, no pre-existent bank account is necessary and the registration can be done via a local mobile money agent. Payment retrieval and deposits such as collecting salaries or welfare payments can also be made through the nearest mobile money agent. This marks a stark change from traditional financial services which were dependent on the existence of a physical financial institution to be accessible (Makina, 2019).

Apart from a lack of reachable physical financial infrastructure, the unbanked population encountered obstacles when attempting to open a bank account, such as a minimum balance requirement, bank charges, and information asymmetries (Aron, 2018; Benami & Carter, 2021). However, mobile money systems circumvent many of these problems by resorting to digital solutions. The solutions are facilitated through developments made by FinTech, which has often been described as “disruptive” in nature (Makina, 2019, p. 302). This is because the absence of credit history previously hindered access to financial products such as micro-loans, behavioral patterns in mobile phone transactions can now suffice to access micro-loans (Benami & Carter, 2021; Gabor & Brooks, 2017). This has led to “leapfrogging” in the area of formal banking services by evading the construction of a large network of physical bank branches (Aron, 2018, p. 135). Nevertheless, certain prerequisites are advantageous for mobile money disbursement such as a baseline level of telecommunication infrastructure, financial literacy, and a reliable energy supply to ensure accessibility (Aker et al., 2016; Aron, 2018; Barajas et al., 2020; Makina, 2019).

Furthermore, research has revealed that mobile money can result in cost saving and greater effectiveness for companies and their employees. Digital distribution of salaries and government payments through FinTech platforms can enhance financial inclusion significantly, with the possibility of increasing private transactions (Aker et al., 2016; Blumenstock et al., 2015; Demirgüc-Kunt et al., 2020). A study conducted by Klapper et al. (2020) in Bangladesh assessed the effect of payroll accounts on the financial inclusion of unbanked Bangladeshi workers. The workers were assigned either a mobile money account or bank account. The study found that the recipients were more likely to have higher account balances, to move savings to formal accounts, increased financial stability, and respond better to income shocks when using mobile money accounts. However, these positive impacts had a stronger effect on people with higher levels of literacy, financial experience, and prior control over finances within the household, which has been supported by further research (Messy & Atkinson, 2013). These individuals were more likely to have a reduced level of consumer protection risk and used the tool more efficiently.

Another finding from Klapper et al. (2020) was that, when being exposed to digital financial tools, the users understood the mechanisms swiftly based on learning-by-doing. This underlines the accessibility FinTech applications offer (Klapper et al., 2020, p. 27). Thus, as Benami and Carter (2021, p.3) demonstrate, mobile money transfers enable a “space-time compression” by exchanging the need for physical financial institutions with mobile phones. Yet, financial inclusion through the use of FinTech such as mobile money can pose risks. Scholars have identified that educational training to increase financial literacy is needed to ensure responsible

usage of FinTech and to create trust amongst consumers in order for them to switch to the new tool (Grohmann & Menkhoff, 2020). Rigid legislation on data security and digital technologies is further required for privacy and consumer protection to be guaranteed, since countries with lower technological development often lack regulatory frameworks. If these aspects are not considered, users can be in danger of information misuse (Benami & Carter, 2021; Demir et al., 2020).

Kenya's M-Pesa system has been the shining example of how impactful mobile money can be for financial inclusion. As Suri and Jack (2016) have concluded in their study, M-Pesa lifted 2% of Kenyan households out of extreme poverty through increased per capita consumption levels. Additionally, the use of M-Pesa has led 185,000 women, previously working in agriculture, to change their occupations to business-related professions. Finally, M-Pesa's success allowed the platform to increase its services by including the distribution of micro-credits, micro-insurance products, and transferring remittances worldwide to M-Pesa users. It must be noted that Kenya provided favorable starting conditions for the distribution of M-Pesa such as implementing a legal framework ensuring consumer protection and market conduct requirements, which allowed for smooth operations (Aron, 2018; Benami & Carter, 2021).

The prominent success in Kenya has accelerated the expansion of mobile money systems to 1.2 billion registered users as of 2020, with many countries adopting the system such as Ghana, Bangladesh, and Uganda (Aker et al., 2016; Andersson-Manjang & Naghavi, 2021; Evans & Pirchio, 2015). Yet, not all mobile money systems have produced similar results in terms of poverty alleviation, such as M-Paisa in Afghanistan. Understanding and evaluating the underlying factors for such differing outcomes is necessary to reduce shortcomings in the future.

3 Historical background

3.1 Timeline of Conflict in Afghanistan since 1979

To understand the current socioeconomic status in Afghanistan one has to understand the historical developments in the country. In the present day, Afghanistan is associated with decades of conflict and failed nation-building. In more recent history, the turning point preceding a period of war was the 1979 Soviet invasion to back the Communist Afghan government. This was followed by resistance from local militia groups known as the Mujahideen, leading to a civil war which ended in 1996 when the Taliban took over power for the first time. Their reign lasted until 2001, when a United States (US) led coalition began its invasion of Afghanistan after 9/11, installing a new government with the goal to restructure the country and promote democracy through large development projects in infrastructure, education, and governance. Yet, on August 15, 2021, the Taliban regained power over the whole country after the withdrawal of all foreign troops (Bloch, 2021; Council on Foreign Relations, n.d.; Rubin, 2013).

The persistence of violent conflicts have had a strong impact on the country by impeding development and economic growth, and resulting in large-scale displacement of persons throughout and outside of the country. Mobile money expansion, in the context of Afghanistan's recent instability, faces novel and extreme challenges such as collective distrust, altered risk-taking attitudes, low investment, technological challenges in terms of infrastructure, and lack of knowledge (Blumenstock et al., 2021; Callen et al., 2014).

3.2 The Hawala System

When analyzing the functioning of M-Paisa, it should be assessed against the backdrop of the Hawala system which has been a pre-existing system shaping the financial sector. The Arabic word *hawala* translates to "transfer" or "trust". The Hawala system in Afghanistan therefore refers to an informal transaction system. It is primarily present in Islamic cultures and characterized by no physical movement of cash, but instead relies on a large international network of Hawala brokers who are in contact with each other. If an individual wants to send money to a recipient, the sender gives the respective amount of money to the Hawala broker present in their region, as well as a unique password specific to this transaction. The recipient then needs to be aware or informed of the password, which they tell the Hawala broker in his region to receive the money. Both Hawala brokers charge a small fee for the transaction and are in contact with each other during the transfer. The Hawala brokers balance and settle their accounts with other brokers in intervals. No paper trail or trace of the transaction is created. The transaction amount varies with no maximum transaction amount set (Chipchase et al., 2011; Fischer, 2005; Maimbo, 2003; Razavy, 2006).

This system is based on one main factor, which is trust between the sender and the Hawala broker. Since Hawala brokers are often sellers or merchants known in their community, the societal pressure and possible exclusion from the community, if the trust of the client is abused, act as guarantees for the system to operate (Fischer, 2005).

While the worldwide Hawala system is efficient, transactions fall outside of formality. Therefore, international transaction fees and taxes are omitted and commonly used for money laundering and other illegal transactions by criminal organizations within Afghanistan (Maimbo, 2003). While it has been linked to illegality, the majority of users utilize the system to support themselves and receive remittances. Decades of war in Afghanistan have deteriorated the financial sector, leading Hawala to be used as a primary transaction method

in the absence of a formal banking system and reviving it as a trusted payment method (El Qorchi et al., 2003; Fischer, 2005; Razavy, 2006).

3.3 State of the Financial Sector in Afghanistan before the Launch of M-Paisa

Due to consistent periods of violence and political instability in Afghanistan, the financial sector was barely existent and in a desolate state. No functioning payment system was in place, with Afghans resorting to the informal Hawala system. Moreover, during the ruling of the Taliban (1996-2001), Islamic views on finance dominated the Afghan financial sector, leading to the abolishment of interest rates by banks and widespread loan defaults. Overall, the sector was characterized by a missing legal framework, lack of qualified personnel, no clear accounting system, and no competitive environment (Fischer, 2005).

The Central Bank (CB) did not fulfill its designated tasks and was rather used by the previous ruling parties for their own benefit. Thus, extensive reforms for the financial sector were necessary, not only for the banks to operate, but also to restart economic activity within the country, improve poverty reduction measures, disburse state salaries and welfare payments as well as develop a reliable infrastructure for a state-wide system and reliable cash flow (Fischer, 2005). In September 2003, a new central bank law reorganized the banking system to be a two-tiered system, since the previous one was outdated and incompatible with a market economy. The Da Afghanistan Bank (DAB) took over parts of the central bank functions, even though the infrastructure did not allow for extensive operations initially (El Qorchi, 2003; Fischer, 2005; Maimbo, 2003).

During the restructuring of the financial sector, the Hawala system was still highly used in Afghanistan due to the previous decline in functioning financial institutions and low level of depositor confidence. Hence, the DAB attempted to regulate this informal financial network to reduce illegal activity, being aware of its importance to the people. Yet, the DAB lacked the extensive resources that would be required to enforce regulations in such a widespread and anonymous system like Hawala (Maimbo, 2003).

Following the reforms of the Afghan financial sector, the main aim of policy-makers and government officials was to create an environment that would foster healthy competition between financial institutions to improve services through market mechanisms, install rigid supervision of the financial sector, and enable economic development (Fischer, 2005). Mobile Money promised to offer such improvements.

4 Methodology

This paper critically examines previously conducted studies and research, while employing various perspectives from development economics, public policy, and international relations. Thereby, the paper aims to engage and build on the existing literature and understanding of mobile money systems as facilitators for financial inclusion from an interdisciplinary perspective. This research uses qualitative methods by analyzing the case study of Afghanistan through a literature review combined with expert interviews. The case study has the advantage of creating an in-depth comprehension of regional specificities and mechanisms. This allows gaining a more reliable understanding to identify the main criteria and/or issues in the case study. The literature review is based on peer-reviewed articles and journals from the digital library from the University of Amsterdam (UvA) and employs a content analysis of the literature. It systematically identifies the most significant factors involved in fostering financial inclusion through mobile money systems, the results mobile money systems have yielded, and the issues linked to them. The papers selected stem from various disciplines such as (development) economics, international relations, finance, digital technology development, and political science. They focus on an interdisciplinary perspective to synthesize the most relevant findings from preceding research conducted in the field and enable the creation of a comprehensive synthesis of the literature, which is still lacking to date. The primary search terms include “mobile money”, “financial inclusion”, “M-Paisa”, “M-Pesa”, and “FinTech”. The papers mentioned and referred to have been published mainly between 2000 and 2021. This time frame is intentionally chosen because of the novelty of mobile money as a whole and the implementation of the system in Afghanistan starting in late 2007. However, there are fewer publications after 2016, which creates the need to synthesize and draw conclusions from other case studies.

Additionally, expert interviews were conducted to add value to this study and support and contrast findings from the literature. The data was gathered using semi-structured interviews, which were substantiated with an interview guide. Semi-structured interviews offer the possibility of having a guiding structure in place, ensuring relevant topics are addressed to create comparability and coherence, while allowing room for follow-up questions, in-depth information, and relationship building with the interviewee (Kallio et al., 2016). A detailed and broadly structured interview guide was created, which was adapted to each interviewee since each had a specialization regarding certain aspects that were covered in the guide. Additionally, due to time constraints of the interviewees, the most important questions linked to their field of expertise had to be prioritized. The interviews with the specific questions tailored to each interviewee allowed for more information to be

gained, and provided an in-depth understanding of key concepts and relationships, especially for the case study of Afghanistan.

The experts interviewed are highly knowledgeable professionals in the field of financial inclusion, mobile money, and development, offering a comprehensive view of the main themes of the paper. The use of this method allows the paper to include primary sources and assess the findings from the literature in comparison to expert opinions. The first interview was conducted with Fatima Popal, the former Director of the M-Paisa system in Afghanistan. She was the Director from 2011 until 2015/2016 in Kabul, and was previously a payment systems consultant at the World Bank. Ms. Popal's in-depth insights were integral to the case study. Her extensive knowledge of the country, the system, and its caveats have substantiated this study immensely. She also enabled access to other experts active in Afghanistan's mobile money through snowballing sampling. This was done via email correspondence to verify data. Secondly, Leora Klapper was interviewed. She is the lead economist at the World Bank and works in the Finance and Private Sector team, as part of the Development Research Group. She is also the founder of the Global Findex database, which provides a substantial part of the data used for this study. She is an expert on financial inclusion, its development, impact, and current obstacles. Thirdly, Martin Cihak was interviewed, who is currently an advisor in the International Monetary Fund's (IMF) Monetary and Capital Markets Department. He worked at the World Bank between 2011 and 2013, leading the Global Financial Development Report, which assesses the advancements made for financial inclusion. Mr. Cihak has been covering financial deepening, financial inclusion, and stability in policy-making. The last interviewee was Tommaso Mancini-Griffoli. Mr. Mancini-Griffoli is currently the Division Chief in the Monetary and Capital Markets Department at the IMF. He has been focusing his work on monetary policy, central banking, and FinTech, which are integral aspects of the development of a robust mobile money system.

All interviewees have a strong academic background and combine first-hand accounts of various mobile money systems with global perspectives on increasing financial accessibility globally, hence allowing for a holistic assessment of the research question.

The ethics guidelines have been followed and no further considerations need to be taken. All interviewees were informed verbally about the purpose of the interviews and were asked for permission to be recorded and quoted in this paper. Consent was given by all interviewees. The interview results are essential for this thesis and will be utilized in many sections contextualizing and assessing deficiencies and achievements made in global financial inclusion through mobile money systems.

5 Analysis: The Case Study of M-Paisa in Afghanistan

Afghanistan's mobile money system launched in 2008 in cooperation with Roshan, an Afghan telecommunication provider, shortly after M-Pesa in Kenya was initiated. As of yet, Afghan M-Paisa has not displayed the widespread success expected from M-Pesa in Kenya. Research has not been able to determine a significant positive impact on poverty reduction, women empowerment, or greater financial inclusion in the whole population, though data is scarce in the field (Blumenstock et al., 2015). The expansion of the mobile money system has been struggling with a multitude of issues, which have not been determined clearly, due to a research gap in the literature. Hence, this analysis aims to identify factors inhibiting M-Paisa's expansion and greater financial inclusion in Afghanistan.

5.1 Initial Conditions for the Introduction of M-Paisa in Afghanistan in 2008

For mobile money expansion, vital pre-conditions have been established in the literature that foster successful distribution and utilization of the service. There are two main areas that need to be evaluated. Firstly, the physical aspect of functioning infrastructure, and secondly, socioeconomic and cultural factors that can influence the mobile money adoption rate need evaluation.

As mentioned before, mobile money can only function if a reliable network infrastructure, energy supply, and telecommunication system is in place (Makina, 2019). A stable political landscape with clear power distribution can allow beneficial government support for the program. An existing regulatory framework for digital payments and cybersecurity ensures smooth operations and trustworthy providers. The level of poverty, education, income, and unbanked individuals affect the chances of adoption of the service and whether a need for the product exists (Aker et al., 2016; Aron, 2018; Camner et al., 2009; Demirgüç-Kunt et al., 2018; Gichuki & Mulu-Mutuku, 2018; Jenkins, 2008).

5.1.1 Mobile and Financial Infrastructure

In 2008, a rudimentary mobile network infrastructure existed in Afghanistan and was expanding in the following years. However, due to unstable energy supply and violence affecting the reliability of the services, network connections varied according to region and time period. Moreover, in areas in which the Taliban were either in control or attacking Afghan government forces and international troops, the cell towers would often be damaged or deactivated for certain periods of time. This would inhibit the usage of the mobile network and

connectivity, creating a problem for mobile money and its attractiveness to the user (F. Popal, personal communication, April 14, 2022). In terms of financial infrastructure, Afghanistan had a very low level of fixed banking infrastructure. Due to a lack of developed basic infrastructure in roads, health, education, and energy networks, further expansion was costly and lengthy (Chipchase et al., 2011; *"Roshan - IFC"*, 2014).

5.1.2 Mobile Phone Penetration in Afghanistan

Mobile phone penetration is one of the most important factors for mobile money, as low financial infrastructure and high mobile phone penetration have been suitable preconditions for successful mobile money rollout (Barajas et al., 2020; Evans & Pirchio, 2015). In Afghanistan, the mobile phone penetration in 2008 amounted to 28 out of 100 people (7.9 million) having a mobile phone subscription. However, in 2011, 46 mobile phone subscriptions per 100 persons were recorded according to the World Bank, marking a 64% growth in subscriptions from 2008 until 2011 (World Bank, 2020b). The market has seen a rapid increase in users amounting to 58 out of 100 persons, or 22.68 million people, with a mobile phone subscription by 2020 (World Bank, 2020b). Additionally, access to a mobile phone network is much higher and was estimated to be above 80% in 2012 (Howell, 2012; F. Popal, personal communication, April 14, 2022). This follows the general trend observed in many low-income countries (Demirgüç-Kunt et al., 2018). Yet, the actual mobile phone penetration is hard to determine due to incomplete data files from telecommunication companies, inaccessible rural areas, and interviewee statements contradicting official values provided by institutions like the World Bank (F. Popal, personal communications, April 14, 2022).

5.1.3 State of the Financial Sector in 2008

By 2008, the formal financial sector was slowly regaining strength following its restructuring in 2003. Assets and deposits at financial institutions were increasing annually by 50% or more since 2006 (International Monetary Fund, 2011). A more competitive environment stimulated this growth, which was fostered by a high number of newly employed public servants and large amounts of foreign aid. According to the Financial Access Survey, 0.3 ATMs existed per 100 000 adults in 2008 on average, demonstrating a higher mobile phone penetration than financial sector outreach (International Monetary Fund, 2021).

The largest shock to the financial sector was the Kabul Bank crisis in 2010 which significantly destabilized the sector. Afghanistan's largest bank declared that $1 billion were stolen through illegal loan schemes. While a total collapse of the bank was prevented through central bank involvement, it highlighted the effects of corruption and poor governance in the country (George, 2020). Even though this occurred two years after M-Paisa's launch, it is crucial since it had a major impact on the stability and trust in the Afghan financial sector (Brown, 2014).

5.1.4 Competition between Mobile Money Providers

By 2008, no other telecommunication provider had launched a mobile money service. This meant that for Roshan, the launch of M-Paisa would not be accompanied by high market access barriers or competitors (Howell, 2012; *"Roshan - IFC"*, 2014). This market advantage was a determinant factor for Roshan to invest and launch the product. Moreover, the facilitating factor of previously established customer ties through the provider has been a key driver for Kenya's M-Pesa system (Camner et al., 2009).

5.1.5 Regulatory Framework

In 2008, a legal framework was established by the Afghan Central Bank regulating and overseeing the implementation of mobile money. Licenses had to be acquired by mobile money providers to safeguard users. While the regulations were extensive and strongly corresponded with M-Pesa's, there were stricter compliance guidelines. One example is that the mobile money provider had to employ all agents directly, preventing the usage of sub-contractors, to create greater supervision and transparency which hampered agent network growth (Camner et al., 2009; *"M-Money Channel Distribution Case - Afghanistan"*, n.d.).

5.1.6 Income Level in Afghanistan

Income has not been established as a significant determinant in fostering mobile money adoption, but high poverty influences many prerequisites for successful mobile money adoption (Patnam & Yao, 2020). The most reliable data from the time frame around M-Paisa's launch outlines that 36% of the population was living below the poverty line in 2008 (International Labour Organization, 2008). The poverty line was defined as an income of $1 per day. In 2008, 28.9% of people in urban areas and 38.2% in rural areas were living below the poverty line, showing a large discrepancy between urban and rural areas (Afghanistan Ministry of Economy & World Bank, 2015). Urban areas benefit from a relatively good infrastructure, service provision, and access to basic social welfare. However, almost 80% of Afghans live in rural areas.

5.1.7 Other Socioeconomic Factors

In Afghanistan in 2008, the literacy rate was 26.2% for all individuals above the age of 15 and 36.5% for 15-24 year-olds (Afghanistan Ministry of Economy & World

Bank, 2015). The net enrollment of children in primary school was 60%. The level of gender equality is very low, which is heavily influenced by religious groups promoting strict interpretations of Islam. These preconditions obstruct females from accessing and engaging with financial services (Howell, 2012). Additionally, corruption was very high, with Afghanistan ranking 176^{th} worldwide out of 180 countries on the Corruption Perceptions Index published by Transparency International (2008).

5.2 Development, Launch, and Outcomes of M-Paisa

For M-Paisa's introduction, Roshan partnered in 2006 with Vodafone Global Services, the same company that introduced M-Pesa in Kenya, to launch a pilot in February 2007, which went public in late 2008 (*"M-Money Channel Distribution Case - Afghanistan"*, n.d.). In 2009, the service was tested by the Afghan police distributing wages to a fixed share of employees producing optimistic results (Blumenstock et al., 2015). An increase of mobile money adoption and higher rates of savings within the group of police employees resulted from the experiment. Many recipients remarked that they initially believed their salary to have increased after receiving their pay digitally. Owing to the high level of corruption in the country, the distribution of wages in cash had interfered with the distribution of the correct wage sum and the digital money transfer had made it much more difficult for corrupt middlemen to pocket a share of the salary (Blumenstock et al., 2015; Chipchase et al., 2011).

This showcases another benefit of the digital salary disbursement: companies significantly reduced delivery costs and decreased time requirements while the employee benefits from greater savings, faster transfers, and higher security (Blumenstock et al., 2015; Chipchase et al., 2011; Evans & Pirchio, 2015). Further evidence of this is provided in Figure 1 which shows that the perceived risk is low in comparison to in-person cash transactions, coupled with low costs, time, and high privacy.

Following the salary disbursement in 2009, M-Paisa expanded its service to airtime transfers, (international) money transfers, cash deposits and withdrawals, bill payments, and humanitarian aid distribution. By 2014, through its agent network, it was present in 230 cities and all provinces (*"M-Money Channel Distribution Case - Afghanistan"*, n.d.). Yet, the usage was concentrated in urban areas close to the major cities as the map in Figure 2 shows, which still holds true today (Blumenstock et al., 2021; F. Popal, personal communication, April 14, 2022).

To become a user of M-Paisa, an individual needs to register with an M-Paisa agent by showing a Government-issued ID and a personal phone number to

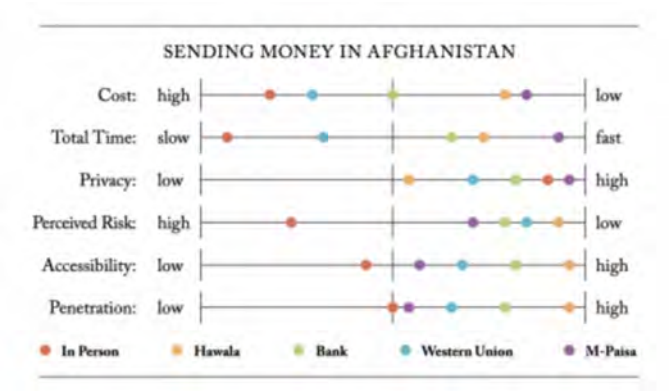

Figure 1: Perception of various money transfers in Afghanistan by Chipchase et al. (2015, p. 45)

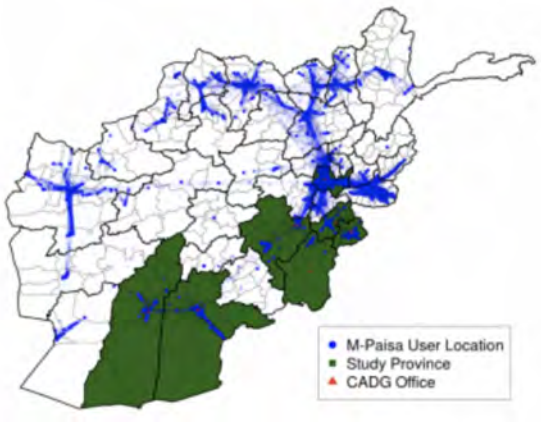

Figure 2: Map of M-Paisa users throughout Afghanistan in 2012 (Blumenstock et al., 2015, p. 2)

which the account will be linked (Suri & Jack, 2011). The rules combat corruption and ghost employees, which have been a frequent issue (F. Popal, personal communication, April 14, 2022; Samuel Hall Consulting, 2014). Per day, a maximum of $2000 can be transferred per person. Per transaction, a fee is charged except when sending money to another M-Paisa user or for withdrawing money from M-Paisa agents. To increase accessibility, an Interactive Voice Response (IVR) system has been created for illiterate users to navigate the application (Blumenstock et al., 2015).

The number of M-Paisa users has seen a steady increase. In 2012, 1.2 million users were registered, and by 2014, 3.5 million users were using the platform (Blumenstock et al., 2021; *"M-Money Channel Distribution Case - Afghanistan"*, n.d.). Additionally, as data from Figure 3 shows, mobile money transactions have experienced an overall positive trend in Afghanistan. The growth in mobile money transactions peaked in 2018 with an average of 438 transactions per 1,000 adults per year. This allows for the assumption that the number of users has been increasing as well. Nonetheless, compared to Kenya, (see Figure 4), where 56,210 transactions per 1,000 adults per year were recorded in 2018,

this marks a huge discrepancy in transaction frequency and overall usage (International Monetary Fund, 2021).

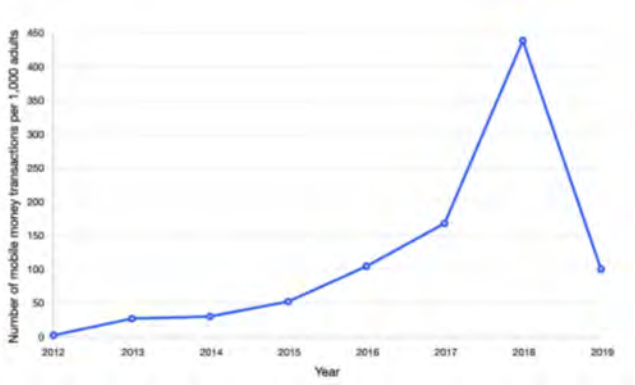

Figure 3: Number of mobile money transactions per 1,000 adults in Afghanistan. Note: Data points were retrieved from the Financial Access Survey from the International Monetary Fund (2021)

It must be mentioned that this indicator does not account for individuals who have multiple mobile money accounts with various providers and can therefore misrepresent the degree of expansion of mobile money (Aron, 2018). In contrast, the Global Findex 2017 recorded 1% of the total Afghan population above the age of 15 having a mobile money account and 2% of all Afghans in the labor force, while 11% had made or received a digital payment in 2017 (World Bank, 2017). While these are two different indicators, they both outline a very frail and shallow integration of M-Paisa in comparison to what was expected.

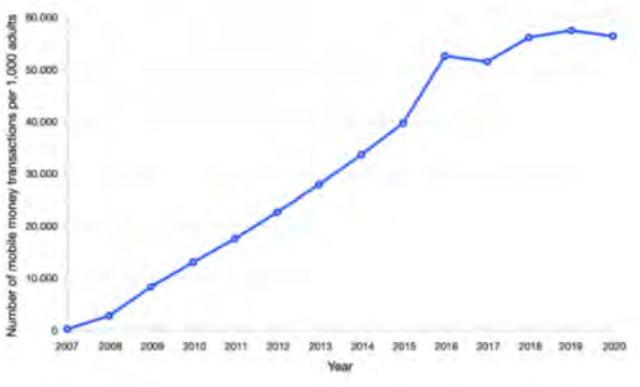

Figure 4: Number of mobile money transactions per 1,000 adults in Kenya Note: Data points were retrieved from the Financial Access Survey from the International Monetary Fund (2021)

This is significant when comparing M-Paisa's increased outreach over time with the exposure and size of other mobile money systems such as M-Pesa in Kenya, or bKash in Bangladesh (Evans & Pirchio, 2015; Herzog & Bolli, 2015; Suri & Jack, 2016).

Evaluating the success of M-Paisa is difficult. Fatima Popal argued that "in the context of Afghanistan, it was moving in a successful direction. Would I say it was where I wanted it to be? No." (F. Popal, personal communication, April 14, 2022). Further, the study by Blumenstock et al. (2015) was not able to detect a significant positive change in the short-run in terms of welfare among M-Paisa users, except for a higher savings rate. Given that the framework is identical to Kenya's M-Pesa system, the results were disappointing.

5.3 Influencing Factors on M-Paisa Adoption and Usage

The following section will attempt to offer insights into the underlying influences that have inhibited greater M-Paisa expansion. Against the backdrop of previously determined initial conditions and obstacles mentioned in the literature, a holistic synthesis of operational pre-conditions will be explored (Aron, 2018; Barajas et al., 2020; Howell, 2012; Suri & Jack, 2016).

5.3.1 Afghanistan's Regulatory Framework

The relationship between a regulatory framework and the success of mobile money systems was investigated by Bahia et al. (2020). They found a significant positive association between an enabling regulatory framework and higher mobile money utilization. This was endorsed by Leora Klapper, who stressed the importance of regulations for the digital finance sector. She outlined that the governments of Nigeria and Ethiopia obstructed the spread of mobile money through their regulatory framework due to heavy lobbying by local private banks. Klapper deduced that "Nigeria is the global outlier in financial inclusions" as a consequence of its inhibiting regulatory framework. Hence, a reliable framework is essential since "to use mobile money you need cybersecurity as the precursor and consumer protection is key for any financial inclusion" (L. Klapper, personal communication, April 12, 2022). This ensures safe usage of the service, especially for individuals that are not well acquainted with digital technology.

Additionally, Martin Cihak pointed out that regulations are not always perceived positively by the population. In Afghanistan, a large part of the informal economy resorts to the Hawala system for payment transactions which functions outside of the legal sphere. Thus, exposure to "taxation on their small business or exposure to other regulations or sanctions" are circumvented, which makes Hawala more attractive than M-Paisa because users do not have to pay taxes and do not contribute to the Afghan economy, which negatively influences M-Paisa usage (M. Cihak, personal communication, April 1, 2022). On the Mobile Money Regulatory Index (MMRI) established by the Global System for Mobile Communications Association (GSMA), Afghanistan ranks

high with a score of 78.81, which should correspond to a high mobile money utilization (see Figure 5) (Bahia & Muthiora, 2019; Bahia et al., 2020).

Figure 5: The Mobile Money Regulatory Index scores per country (Bahia & Muthiora, 2019, p. 4)

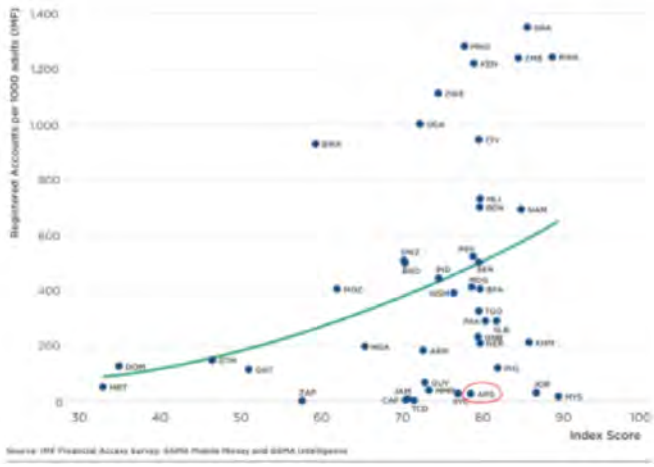

Figure 6: Regulatory Index scores and mobile money adoption (Bahia & Muthiora, 2019, p. 9) Notes: Relationship between MMRI and registered mobile money account per 1000 adults

Yet, as Figure 6 indicates, the high MMRI Index score does not result in a high level of registered accounts, differing from the associations found by Bahia et al. (2020). This is confirmed by Fatima Popal, who remarked

> "I would not say the government promoted it [M-Paisa] but they allowed it. [...] They would never specifically forbid mobile money but definitely could have pushed for it much stronger," (F. Popal, personal communication, April 14, 2022)

highlighting the lack of active support by the Afghan government.

Moreover, as Evans and Pirchio (2015) emphasized, the government decisively influences if the mobile money system will be prosperous depending on the promotion of regulatory frameworks. A regulatory framework which does not include commercial banks as vital players for mobile money and which has a low level of restrictions in place is a determinant of success. Consequently, these findings suggest that the framework itself in Afghanistan is robust and not hindering the expansion of mobile money, but country-specific factors such as a lack of support from the political leaders, for example in the form of policy-making, impeded the adoption of the service.

5.3.2 Trust in Financial Institutions

Another factor inhibiting M-Paisa expansion is the lack of trust in formal institutions. Globally, insufficient trust is one of the main reasons for individuals not to adopt mobile money (M. Cihak, personal communication, April 1, 2022; L. Klapper, personal communications, April 12, 2022). According to EY's 2021 NextWave Consumer Survey, FinTech is the worldwide most trusted financial service by all age groups except for those aged 65 years and above (Lee & Mannamkery, 2021). Klapper adds that "there is the perception that it's the markets people trust, hence the FinTechs more than the banks," which should infer high acceptance and adoption within the Afghan population (L. Klapper, personal communication, April 12, 2022).

However, in Afghanistan, the scarce presence of financial institutions paired with incidents such as the Kabul Bank crisis and political instability have fuelled low trust in financial institutions, which acts as a barrier to higher financial inclusion (Blumenstock et al., 2021; Chipchase et al., 2011; International Monetary Fund, 2011). In this case study, the lack of trust has influenced M-Paisa, despite being a trusted FinTech. This is supported by Leora Klapper's statement that "having had banking crises in the last 20 years affects the level of trust heavily." (L. Klapper, personal communication, April 12, 2022).

Martin Cihak points out another consequence of the lack of trust in fragile countries: "voluntary exclusion", referring to actively abstaining from using (digital) financial services (M. Cihak, personal communication, April 1, 2022). This process acts in opposition to programs fostering greater financial inclusion. Yet, Cihak distinguishes between short-term and long-term transactions, arguing that sending money has a higher prevalence than savings since less trust is necessary. For savings, the user needs to trust that their money is stored safely and is accessible, which is not needed with fast transactions. In Kenya, "people preferred to use M-Pesa for transactions and not necessarily for savings," which supports the claim that lacking trust was inhibiting greater usage. Additionally, the fact that individuals already use mobile phones for daily activities leads to a

higher inclination to trust services using the same device (M. Cihak, personal communication, April 1, 2022). This can be applied to Afghanistan since the savings rate is low for M-Paisa accounts while transfers are more frequent (Blumenstock et al., 2015).

Fatima Popal confirmed that trust was a challenge to M-Paisa since users had problems “with the concept of what happens when my money gets deposited electronically and where does it physically go [...] making it harder for people to trust M-Paisa” (F. Popal, personal communication, April 14, 2022).

5.3.3 Impact of Violence

Another aspect that can provide insight into the low M-Paisa usage is the impact of violence. One of the main advantages of M-Pasia is the higher level of security when saving, transferring, or receiving money. The security hazard of storing physical cash which can be stolen or lost during transport is resolved (Aron, 2018; Blumenstock et al., 2015; Cihak et al., 2021; Demirgüç-Kunt et al., 2018). Countries with a low level of income and poor financial infrastructure such as Afghanistan provide favorable conditions to mobile money providers since they can resolve a large amount of economic friction due to a lack of alternative services (Evans & Pirchio, 2014; Makina, 2019).

In Afghanistan, the level of violence was consistently high in the last decades (Callen et al., 2014; Blumenstock et al., 2021). Following the previously established relationships, one would assume that Afghanistan has a high adoption rate for mobile money due to a country profile distinguished by low-income, poor (financial) infrastructure, and instability (Evans & Pirchio, 2015). However, in the case of Afghanistan, Blumenstock et al. (2021) points into a different direction and suggests that violence and the resulting risks have been detected as an inhibitor for mobile money adoption .

Firstly, in a field experiment assessing the welfare effects of M-Paisa on users, the participants did not experience a higher level of physical security through M-Paisa (Blumenstock et al., 2015). A second study by Blumenstock et al. (2021) assessed the effects of violent events on M-Paisa users in the time frame 2010-2012. The results highlight a strong negative relationship between exposure to violence and M-Paisa usage: the study participants reduced their balance accounts and transactions when the level of violence increased in their environment. These changes were consequences of the expectation of future violence to occur. The advantage of physical cash providing constant liquidity outweighed mobile money. Even security concerns connected with cash such as theft and loss did not prevail over future violence concerns. A network effect can be perceived here as the value of cash increases since the utility of cash improves due to higher violence (Callen et al., 2014; Chipchase et al., 2011; Murendo et al., 2018). Fatima Popal points out that violence reduced M-Paisa’s reachability, which is intensified by the difficulties recruiting agents in areas with high violence. She explains that:

> "[When] Turning off telecom cell towers in specific areas or at night, coverage is not given and hence mobile money does not work. If there is an area that is fighting and insecure, that area would be more difficult to bring in mobile money and have people use it due to the fact that it is not usable" (F. Popal, personal communication, April 14, 2022).

Thus, the country-specific high frequency of violence experienced by the population acts as an obstacle and stands in contrast to general mobile money usage and its ability to surpass financial development stages through leapfrogging (Aron, 2018). Hence, the future expectation of violent events paired with a low diffusion of M-Paisa agents intensifies the reduction in usage of the service (Blumenstock et al., 2021; Evans & Pirchio, 2015).

5.3.4 Hawala as a Competitor

Hawala is a fixed component in the Afghan financial landscape owing to its long-standing reputation, reliability, anonymity of the user, and widespread presence (Maimbo, 2003). M-Paisa had to convince regular Hawala users to switch and/or adopt M-Paisa, however with limited success. Thus, the assumption was, in line with contemporary literature, that Hawala acts as a competitor for M-Paisa (Blumenstock et al., 2015; Chipchase et al., 2011; Howell, 2012).

Martin Cihak confirmed that Hawala’s role as a competitor negatively influenced the demand for M-Paisa. He outlined that the main selling points of mobile money depend on their alternatives. In Afghanistan, given the difficulty of accessing the banking system, the convenience factor is decisive for individuals to determine which service to use. While M-Paisa yields higher convenience, it is not as established as Hawala, which impedes its adoption. Moreover, in light of the ongoing humanitarian crisis which started with the departure of the US troops in 2021 , Cihak points out that “they seem to be turning back to the Hawala system, but also rely more on cryptocurrency and bitcoin as payment methods to avoid sanctions and taxes”, emphasizing the perpetual need for Hawala (M. Cihak, personal communication, April 1, 2022).

Contrastingly, Fatima Popal remarked that “Hawala were not exactly competitors for us,” refuting the previous claims. Rather, M-Paisa approached Hawala brokers to recruit them as M-Paisa agents

> “since they had liquidity, [and] mobile money is dependent on agents having access to available cash. Especially because

> the agents were already known in their community and have a following, they proved to be trusted agents for M-Paisa" (F. Popal, personal communication, April 14, 2022).

Due to the scarce financial infrastructure in the country, M-Paisa had to provide liquidity to the users and needed agents capable of transferring, as well as distributing, large sums of money on the spot to be successful. This demonstrates an opposing view, as it suggests that Hawala could be an attribute rather than a competitor, which has not been mentioned in the literature and could indicate that false assumptions of Hawala being too strong of a competitor were used as explanations for why M-Paisa was not successful.

Consequently, this presents a lack of coherent and recent information regarding Hawala's influence on M-Paisa. Yet, the country-specific premise of a well-developed, informal transaction system like Hawala induced unique developments in Afghanistan's financial sector which was not seen in other countries such as Kenya.

5.3.5 Socioeconomic Conditions

Mobile money as a driver of financial inclusion has been launched in many low-income countries with unique cultural and socioeconomic factors which can inhibit or foster the adoption of the product (Barajas et al., 2020; Blumenstock & Eagle, 2010; Evans & Pirchio, 2015). The case of Afghanistan demonstrates extreme socioeconomic levels in comparison to other low-income developing countries, questioning their effect on M-Paisa.

Although literacy is no pre-condition to operate M-Paisa, a basic level of education, and especially financial literacy, enhances effective usage of mobile money (Demirgüc-Kunt et al., 2018; Howell, 2012; Klapper et al., 2020; Suri & Jack, 2011). In 2021, only 37% of all Afghans above the age of 15 were literate, while poverty in Afghanistan was highly prevalent (World Bank, 2021). Research found that a higher level of education enhances awareness and adoption of mobile money services. Furthermore, female involvement in enterprises, finances, and decision-making increases this likelihood (Guchuki & Mulu-Mutuku, 2018; Suri & Jack, 2016). Hence, the low level of education paired with little to no financial resources to foster education does not increase the uptake of mobile money services. Consequently, M-Paisa adoption was not enhanced by these socioeconomic factors.

According to Fatima Popal, education was the biggest obstacle for M-Paisa, confirming this claim. Even though trainings and workshops were implemented, she underscored that due to different education levels, especially in rural areas,

> "the teaching of the usage was already almost too much. If someone was not able to read or write, explaining to them [how] to use the voice interactive service was very difficult." (F. Popal, personal communication, April 14, 2022).

Additionally, a worldwide gender gap is present with mobile money usage, also applicable to Afghanistan (Demirgüç-Kunt et al., 2018; Hess et al., 2021; Lee et al., 2022). Reasons for this are multifold in Afghanistan such as "in some of the families only the men owned a phone, which was most common in the rural areas versus in the cities" (F. Popal, personal communication, April 14, 2022). Financial inclusion refers to equal ownership and usage of financial services by men and women; Klapper, however, points out that "Afghanistan [...] is one of the countries where men are 4-5 times more likely to own a mobile phone than women. Hence, women will be left behind at the starting gate if they do not have access." (L. Klapper, personal communication, April 12, 2022). She extenuates the structural issues present in Afghanistan linked to unequal gender access to mobile phones and therefore M-Paisa adoption.

Moreover, female interaction with unknown males in the form of M-Paisa agents is problematic due to religious and cultural factors (Howell, 2012). Klapper et al. (2020) conducted a study in Bangladesh, where women were increasing their bank machine usage but not mobile money. The reason for this was that:

> "Women were uncomfortable giving a male agent their mobile phone number to activate their account and hence, refrained from using the service fully. The chance/risk of being harassed was too high to engage with the agents. This is a huge barrier but at the same time a great chance for more inclusive mobile money" (L. Klapper, personal communication, April 12, 2022).

Given Klapper's synthesis from her research in Bangladesh, it is possible that similar factors prevent women from participating in mobile money in Afghanistan, given the gender inequalities that persist. M-Paisa attempted to increase the number of female agents to circumvent obstacles linked to gender. Nevertheless, to what extent gender influenced M-Paisa usage cannot be deduced due to a lack of research.

Furthermore, since 99.7% of the population identified as Muslim, the religious influence of Islam on mobile money in the form of Islamic Finance in Afghanistan had to be incorporated by M-Paisa. Islamic Finance outlines financial practices in accordance with Sharia, the Islamic law. This mainly concerns risk-sharing in investments. Investors are rather co-owners instead of creditors, taking on any losses and profits. Islamic Finance objects to certain common financial practices involving investments in sectors such as the alcohol or arms industry, and the charging of interest since this creates wealth for the lender from capital only. In 2017, 14%

of Afghans indicated that they did not have any type of account at a financial institution due to religious reasons, pointing towards voluntary financial exclusion as a cause of financial services not incorporating Sharia law into their practices. M-Paisa's transaction fees are sometimes mistaken for interest rates, which have led to issues with Islamic Finance compliance (Hanif, 2016; *"M-Money Channel Distribution Case - Afghanistan"*, n.d.; World Bank, 2017).

Consequently, the interplay of these socioeconomic factors negatively influenced M-Paisa dispersion. Fatima Popal underlines this by comparing socioeconomic factors in Kenya to Afghanistan: "The levels of Afghanistan versus Kenyan levels were very different. While we did take M-Pesa's framework and remodel[ed] it for Afghanistan, there was a lot more that needed to be done in Afghanistan due to its stage of development." (F. Popal, personal communication, April 14, 2022). Ultimately, this showcases that Afghanistan's mobile money expansion did not develop as fast and easily as it was the case in Kenya.

5.3.6 Active versus Registered Mobile Money Users in Afghanistan

Understanding the behavior of mobile money users is vital to evaluate the success of a mobile money system. As Hamdan et al. (2021) have pointed out, not all registered users are active users. When an individual acquires a registered phone number, network providers operating mobile money systems simultaneously create a mobile money account. Hence, the amount of mobile money users possibly distorts the dimensions of mobile money usage. As seen in Figure 7, a large gap is visible between registered and active users of mobile money services in Afghanistan.

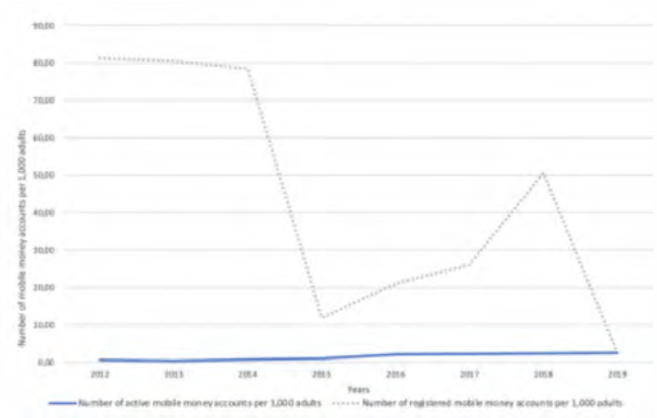

Figure 7: Registered and active mobile money users per 1000 adults in Afghanistan Note: Data points were retrieved from the Financial Access Survey from the International Monetary Fund (2021)

Firstly, this gap has been changing strongly from year to year without an indicated reason, perhaps due to the incoherence and difficulty of obtaining data in the country. Secondly, a stark reduction of active and registered users has been reported in 2019, resulting in 2.53 active and 2.53 registered users per 1000 adults in Afghanistan (International Monetary Fund, 2021). The fact that the numbers are identical could be another indicator of a lack of detailed data sets.

While no reason for the extreme reduction in transactions after 2018 has been provided in the literature, a possible explanation is that in 2019 a high number of telecommunication towers were destroyed by the Taliban and other insurgent groups. As Tolo News, the largest media channel in Afghanistan reported, over 220 towers were demolished within the year 2019 (Nikzad, 2019). Due to Afghanistan's mountainous landscape, the density of cell towers has to be higher to provide coverage. Without any functioning cell towers due to destruction in the M-Paisa user's environment, no usage is possible (Howell, 2012). If no reachability of the service is guaranteed, expansion is harder, since users expect accessibility at all times. Only if the benefits of M-Paisa in comparison to Hawala are guaranteed, M-Paisa has an advantage on the market (F. Popal, personal communication, April 14, 2022).

Another aspect linked to the decrease in active mobile money users is the scope of the agent network in Afghanistan. For M-Paisa to function, it is paramount for the service to be present in the form of agents. A lack of M-Paisa agents reduces the possibility for users to access the service and be active (Evans & Pirchio, 2015). According to data by Roshan from November 2021, there were 1,011 M-Paisa agents in Afghanistan, but barely any agents in rural regions. The current liquidity crisis in Afghanistan decreased the availability of cash further, since the DAB has no access to the international banking system following the Taliban takeover in September 2021 ("Afghanistan: Economic Roots of the Humanitarian Crisis", 2022; F. Popal, personal communication, November 16, 2021). This not only decreases the impact of M-Paisa but limits the understanding of participation and its effect on the mobile money system (Aron, 2018; Hamdan et al., 2021).

5.3.7 Importance of Scale

For mobile money providers to be profitable, scale is necessary (Cobert et al., 2012; Donovan, 2012). Firstly, being profitable is important to sustain independence, for example, if grants by, e.g., governments or international investors end. Secondly, profit is required for investments in new technologies (Osafo-Kwaako et al., 2018). While M-Paisa was funded partially through foreign aid, reaching scale is vital for its continuation and development when these funds stop, which is the case since the Taliban came to rule in 2021 (Howell, 2012; Malikzada, 2022).

Martin Cihak emphasizes that "especially the Hawala broker system had a great influence on the expansion

of mobile money, making it harder for M-Paisa to gain scale in the country compared to some East African countries", indicating the challenges of M-Paisa within the Afghan market due to lower demand because of Hawala's presence (Brown, 2014; M. Cihak, personal communication, April 1, 2022).

Evans and Pirchio (2015) support the view that mobile money systems need to reach a certain number of users (critical mass) in a given time frame for the service to be successful, arguing the greater the improvement for individuals through the usage of the service, the higher the number of users. Afghanistan's fluctuation in user growth and number of transactions (see Figure 3) shows that the critical mass was not reached for exponential growth to take place. Internal documents provided by Fatima Popal substantiate that M-Paisa has not been profitable since its launch and is currently sustained through profits made by Roshan (F. Popal, personal communication, November 26, 2021). As a result, M-Paisa's lack of scale contributed to the comparably low number of users throughout the country, handicapping further investments.

5.4 Digression: Mobile Money in Kenya - Factors of Success

To be able to classify M-Paisa's disappointing results, it is useful to take a look at M-Pesa in Kenya: a mobile money system that has set a benchmark for systems elsewhere. The mobile money system experienced unique rapid growth through high adoption rates and active usage (Suri & Jack, 2011). Two years after its launch, M-Pesa recorded 7.7 million users, over 10 000 agents, and 65% of households using the service in 2009 (Suri & Jack, 2011, 2016). The critical mass was reached fast and allowed for scale, expansion, and profitability (Brown, 2014; Creemers et al., 2020; Evans & Pirchio, 2015; Muthiora, 2015).

At the time of M-Pesa's launch in 2007, high demand for a convenient, fast, and safe method of money transfer was present since 58% of the Kenyan population was transferring money by hand the year before (Camner et al., 2012). Moreover, no dominant informal money transfer network was in place to compete with the service, as was the case with Hawala in Afghanistan. In 2007, Kenya displayed a well-functioning infrastructure, a 72% literacy rate of adults above the age of 15, and a mobile phone penetration of 48 out of 100 people having a subscription (Suri & Jack, 2011; World Bank, 2021). Lower rates of violence supported mobile money adoption instead of impeding it because a safer transfer and savings platform were beneficial to the users given reliable accessibility (Blumenstock et al., 2021; Suri & Jack, 2011).

While there was no regulation in place before the launch of M-Pesa, the central bank actively supported M-Pesa by providing enabling regulations and a legal framework fostering M-Pesa adoption. Their score of 79.24 on the MMRI, which is comparable with Afghanistan's score of 78.81, correlates with the high number of transactions in contrast to Afghanistan. Thus, the outcome of M-Paisa is attributable to exogenous factors, instead of to the robustness of M-Pesa's framework (Bahia & Muthiora, 2019). Moreover, Kenyans were familiar with basic technologies facilitating the adoption and usage of the digital service (*"M-Money Channel Distribution Case - Kenya"*, n.d.).

Furthermore, M-Pesa was launched by Safaricom, the largest mobile network provider in Kenya with a 77% market share (Aron, 2018; *"M-Money Channel Distribution Case - Kenya"*, n.d.; Suri & Jack, 2011). Hence, a higher number of users of the same system were guaranteed, coupled with the option to send payments to non-M-Pesa users. Therefore, the convenience of mobile money was guaranteed (M. Cihak, personal communication, April 1, 2022). Additionally, 35% of Safaricom is owned by the Kenyan state, suggesting a generally positive attitude to the expansion of mobile money (*"M-Money Channel Distribution Case - Kenya"*, n.d.). This is a stark contrast to Afghanistan, where Roshan's share, even though it was the largest mobile operator, was much smaller and the market was divided into four main operators. In addition, since mobile providers in Afghanistan are linked to prominent families, adoption of services depends on the owner's ethnic or religious background. This operated as a barrier to the growth of mobile money (Brown, 2014; F. Popal, personal communication, April 14, 2022).

General consensus in the literature is that the aggregator model of mobile money agents strongly enhanced M-Pesa's success. This model outsourced agent recruitment to a restricted number of M-Pesa agents, which were employed directly by Safaricom. This allowed faster growth of the agent network, using the regional knowledge of the Safaricom agents to recruit trusted sub-agents within the communities (Camner et al., 2009). The importance of an expansive agent network becomes apparent in Suri and Jack's (2016) analysis, which found that poverty reduction and extreme poverty in Kenya were linked to higher mobile money agent accessibility. However, due to the restricting conditions set by the Afghan Central Bank, this model could not be introduced in Afghanistan, impeding agent network expansion (*"M-Money Channel Distribution Case - Afghanistan*", n.d.).

When asked if Popal believed that M-Paisa's poor usage is due to a possible lack of robustness in the framework mirroring M-Pesa in Kenya, she replied:

> "[It is] definitely country-specific. Due to education, government, technology, coverage, phone coverage, marketing, advertising. The levels of Afghanistan versus Kenyan levels were very different. While we did take M-Pesa's framework and remodel it

> for Afghanistan there was a lot more that needed to be done in Afghanistan due to its stage of development" (F.Popal, personal communication, April 14, 2022).

Klapper substantiates Popal's argument by remarking that

> "I perceived mobile money to just take off in Afghanistan. Crime and safety have a unique influence in Afghanistan since it is safer to store your money outside of the home than inside and traveling with cash is very dangerous. This is why it took off in Somalia and other conflict zones because it provides a greater degree of safety." (L. Klapper, personal communication, April 12, 2022).

In addition, Martin Cihak asserts that M-Paisa did have "life-changing potential" but acknowledges at the same time that "Afghanistan has always been complex, even before the second Taliban reign" (M. Cihak, personal communication, April 1, 2022).

6 Discussion

6.1 Policy Suggestions

Resulting from the research for this study, policy suggestions for M-Paisa could be made. Firstly, an increase of female mobile money agents could contribute to a more widespread usage, and secondly, introducing an integrated mobile money platform in Afghanistan would reduce inefficiencies and improve the scale of the service. Since M-Paisa has been struggling to gain scale, these findings should encourage further research into the mobile money system in Afghanistan.

6.1.1 The Importance of Female Agents

A common theme identified by interview partners Leora Klapper and Fatima Popal is a greater effort to foster financial inclusion. As Buvinic and O'Donnel (2019), Demirgüç-Kunt et al. (2018) and Howell (2012) argue, women face greater challenges when accessing financial services and taking advantage of economic opportunities. Thus, female agents, which enable female-to-female interactions, have to become an integral part of the mobile money agent network, which needs to be included in the framework.

The fact that women have refrained from registering for a mobile money account due to fear of harassment from male agents is far-reaching and underlines the importance of female agents. Leora Klapper calls attention to this since she believes that "[gender] inclusiveness is a very large barrier for mobile money in Afghanistan" (L. Klapper, personal communication, April 12, 2022). With higher prevalence of female agents, religious hurdles such as interactions between unknown men and women could be surmounted. Furthermore, access to economic opportunities for women is hampered by conservative values stemming from the prevalence of religious extremism in Afghan society (Rahman, 2018).

Studies have concluded that female-headed households allocate more resources to children or goods that improve their well-being such as nutritious foods than male-headed households, though education and urban or rural residency impact the amount of resources being allocated. Moreover, women tend to invest more of their time towards the upbringing of a child within the household than men (Bose-Duker et al., 2020; Caiumi & Perali, 2015; Duflo & Udry, 2004). Thus, if greater female usage of mobile money with the help of female agents can be achieved, the feats for poverty reduction resulting from financial inclusion could be enhanced. This link is based on the positive effect of M-Pesa usage in Kenya which was able to reduce the extreme poverty level (Suri & Jack, 2016).

Even though female agents were employed in Afghanistan, this was not on a general basis, as Fatima Popal explains:

> "Female agents were project-specific, especially for MFIs where all of them were women and the trust was larger. [...] However, it was harder to find women available as agents as well as women using M-Paisa due to a lack of education and career opportunities" (F. Popal, personal communication, April 14, 2022).

This neither encourages nor opposes a larger share of female agents in mobile money systems such as M-Paisa. It rather outlines the unique hurdles in Afghanistan (e.g. education, recruitment, and implementation) that interfered with more extensive female involvement. Nevertheless, it suggests that more female agents could facilitate female integration in financial decision-making by reducing the threshold for females to use M-Paisa in Afghanistan.

6.1.2 The Integrated M-Paisa Platform in Afghanistan

Another aspect brought up by Fatima Popal is the importance of interlinking different telecommunication providers and their mobile money services. Following M-Paisa's launch, other telecommunications companies initiated their own mobile money products. While they did not reach the size of M-Paisa, they are competitors in the market. However, the various providers do not allow for transfers between the different systems. This causes inefficiencies and hampered accessibilities in the market, reducing consumer satisfaction and convenience.

Since mobile money in Afghanistan was not primarily promoted to create new financial products for private

companies, but rather to reduce the share of the unbanked population in Afghanistan and to offer safer and more accessible transaction and savings options, an integrated platform should be created to increase the benefits for development. Fatima Popal stresses its necessity by emphasizing that:

> "In order for this to excel more, all the telecom companies need to work together. They need to use the integrated platform that was created for them. The government needs to be on the same page and accept M-Paisa/mobile money for the transparent systems it does create" (F. Popal, personal communication, April 14, 2022).

Additionally, Popal remarks that "many of the [international development] organizations do not know about M-Paisa which needs to be changed for them to use the service to distribute their salaries via this product" (F. Popal, personal communication, April 14, 2022). Little awareness and support by the government has not scaled M-Paisa enough in the last years to be used in new sectors within the country. However, this is necessary for it to be dispersed on a greater level and become essential for more Afghans. In line with previous suggestions, mobile money expansion is more viable the greater the scale (Cobert et al. 2012; Evans & Pirchio, 2015). Currently, favoritism towards the traditional banks is halting more companies to use M-Paisa for activities like salary disbursements even though the service offers high transparency and thus is better at combating corruption. At best, policy-makers should keep these specific suggestions in mind that have been determined to have potential in facilitating and increasing M-Paisa's usage and success.

6.2 Limitations

While the case study of Afghanistan offers great insight into the problems mobile money systems encounter in fragile countries, limitations are apparent and need to be addressed. Firstly, the majority of this paper analyzes a loose time frame between 2008 and 2019. This time frame was characterized by relative stability in terms of governance and development but had constantly high levels of violent events, conflict, poverty, and corruption. However, the Taliban takeover in September 2021 has caused great turmoil and fear among the international community that attempts for development and stability in the country will be demolished. This ranges from improving educational levels and human rights advocacy, especially for women, to basic infrastructure development and economic projects with international partners (Malikzada, 2022; Padshah & Goldbaum, 2022; Zucchino & Padshah, 2022).

The effect of the Taliban rule on mobile money is not clear yet. Information is either inaccessible or has not been recorded. Since international funding was cut off after September 2021, M-Paisa has been struggling to sustain itself due to a lack of profitability. If M-Paisa does not experience significant growth and if international funding will not be reinstated, the future of M-Paisa is not guaranteed. To what extent Sharia law will impact the usage of the service is unclear as well. The Taliban did not interfere previously with mobile money expansion, usage or M-Paisa directly, which could suggest little influence on its usage (F. Popal, personal communication, April 14, 2022). A larger issue is that M-Paisa has to ensure that its service will not be abused for illegal transactions. Only if it will be able to continuously operate and monitor activity in the whole country, can this be guaranteed.

Nevertheless, the potential for FinTech and development in low-income countries is high. Tommaso Mancini-Griffoli argued that by decreasing the cost of transactions, the cost of access is decreasing and, in return, this increases reachability. Hence, FinTech would alter the financial sector in any case (T. Mancini-Griffoli, personal communication, April 30, 2022). Another aspect is that due to data insufficiency, clear causalities and trends were not able to be determined. While Fatima Popal offered essential insight into country-specific aspects, greater exposure to first-hand sources such as employees at M-Paisa and users of M-Paisa would have strengthened the analysis. This has obstructed an extensive future outlook for M-Paisa in this study. To verify the reasons for the low levels of M-Paisa usage and transactions in comparison to Kenya with more certainty, more consistent, accessible, comparable, and in-depth data sets need to be compiled.

This applies also to the effect of the COVID-19 pandemic on Afghanistan and its mobile money services. As the World Development Report 2022 analyzed, the pandemic led to a stark increase in digital transactions and uptake of digital financial services. However, regional discrepancies exist. Data from Pakistan, a country neighboring Afghanistan, indicates that the higher uptake of digital financial services is most prevalent in urban areas with individuals possessing financial literacy (World Bank, 2022). How the pandemic affected digital financial services in Afghanistan is not yet documented, highlighting the necessity for additional research.

7 Conclusion

Mobile money has shown to be a driver for financial inclusion in politically fragile countries where traditional financial infrastructure has not been effective. Kenya's M-Pesa system has highlighted the potential achievements mobile money can unravel for greater usage of financial services, poverty reduction, and financial stability. The case of Afghanistan, on the other hand, has emphasized another key understanding: the importance of incorporating country-specific conditions into the respective mobile money systems. While Afghanistan's M-Paisa system initially experienced growth, the expansion and rate of transactions have not reached the level of M-Pesa, even though the same framework was used.

Concluding from the literature review and expert interviews, the framework of M-Pesa has been deemed robust (O'Rourke, 2007). It entails a sturdy system and provides a strong set of regulations that enable mobile money growth. The low level of M-Paisa prevalence can therefore rather be attributed to country-specific factors that hindered the wide adoption of the system. The analysis suggests that there were multiple countervailing, exogenous factors. These were the low technological development and infrastructure, lack of trust in financial institutions, socioeconomic factors, a constantly high level of violence, lack of scale, and competition with the Hawala network. The framework was not able to account for these inhibiting factors alone. While mobile money as a tool to foster economic development creates potential for leapfrogging, in the case of Afghanistan, the extreme nature of the country obstructed reaping the benefits of greater financial inclusion through mobile money.

The issues identified hampering the expansion of M-Paisa can be applied to other regions with similar conditions and important lessons can be drawn from M-Paisa in Afghanistan. Yet, the drastic and country-specific social, cultural, and development factors do not allow for generalizations.

Consequently, this paper contributes to the understanding of mobile money in Afghanistan, the country-specific issues impeding expansion, and the differences in comparison to M-Pesa in Kenya. It outlines recommendations for future research and policy-makers such as the importance of female agents and using an integrated platform for all mobile money providers, which can be helpful to other countries with similar issues of gender inequality in FinTech usage.

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Social Sciences

Disappearing through the Border

Narrating the *Desaparecidos* through the Argentinian Diaspora in Europe

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Abstract

This study aims to answer how the Argentinian diaspora in Europe narrates their identity in relationship to their shared traumatic past. The latter concerns the political oppression of the last military dictatorship and the advent of the *desaparecidos*. This research is based on a theoretical background of socio-cultural trauma and collective memory and investigates the main narratives that developed in Argentina during and after the last dictatorship, which were then used as the analytical framework for the qualitative study that was later carried out. The latter included five participants, all members of the Argentinian diaspora in Europe, aged from 43 to 69 years old. Semi-structured interviews produced qualitative data that was then analyzed and categorized into thematic units. The results of the study show that members of the diaspora prefer framing the events of the *desaparecidos* as state terrorism. These perspectives are shaped by their individual life experiences and exposure to the different discourses that interpreted this part of history. Moreover, although the interviewees recognize the way Argentinian society has been impacted by the events of the last dictatorship, their national identity did not seem particularly shaped by it, relying instead on more recent occurrences in Argentina and its internal struggles.

Keywords and phrases: *Argentina, desaparecidos, trauma, memory, identity*

Suggested citation:
Scribani A. (2022). Disappearing through the border: Narrating the *Desaparecidos* through the Argentinian Diaspora in Europe *AUC Undergraduate Journal of Liberal Arts and Sciences*, 19(1) pp. 86-105

imprint.

Contents

1 Introduction

> "Not scars. They really are open wounds, not scars. Why? Because when you're still looking for an answer that does not arrive, the scar can never form".
> – Valentina (52, Argentine living in Italy)

Since its independence from Spanish colonial rule in 1810, Argentina's history has been demarcated by civil wars, dictatorships, and state repression. The most recent violent period was the military dictatorship under the generals Videla, Viola, and Galtieri, which took place between 1976 and 1983 (Romero, L. A. & Brennan, 2013). A fundamental characteristic of this autocracy is the scale and means of political repression, which marked history as the case of the *desaparecidos*. The "disappeared", political opponents that acted against the regime both as members of armed guerrillas or as civil resistance, involve 30,000 Argentines that were strategically abducted, subjugated, tortured, put in "rehabilitation camps", and mostly killed by their own government (Robben, 2018). These atrocities resulted in families desperately searching for their disappeared relatives, mobilizing street protests, and re-organizing in support groups such as the Mothers and Grandmothers of Plaza de Mayo (Vezzetti, 2003).

Understandably, these extreme events have had a tragic impact on Argentinian society, leaving what many scholars refer to as a "large-scale trauma" (Robben, 2018). What makes the Argentinian case particularly relevant is how the events have been interpreted since the end of the dictatorship in 1983. The survivors of state violence and the relatives of the victims faced a laborious succession of different narratives about what had happened to them: from complete denial of the events, to being recognized as justified war means, to being acknowledged as state terrorism and finally genocide two decades later. These different narratives shaped Argentines' memories and affected the way the dictatorship is perceived and remembered today. In Argentina, the memory of the *desaparecidos* and of the atrocities that took place is kept alive through memorials, monuments, national remembrance days, and other civil society processes. However, for the Argentines who do not live in their home country anymore collective memories are harder to access, therefore individual memory takes on a different role as the main bearer of recollections and is therefore a fundamental definer of national identity. Whilst a significant body of literature analyzes the working mechanisms of collective remembrance in Argentina, almost no research has been conducted around the role that these events play in the shaping of Argentines' identity, especially when living abroad.

In this research, memory – and the way narrative operates through it – will be the conceptual tool to study the way the Argentinian diaspora in Europe recounts their history. The following research question will be explored: *How does the Argentinian diaspora in Europe narrate their identity in relationship with their shared traumatic past?*

This study provides an insight on how large-scale violence impacts entire societies; not only beyond the geographical area where the events took place, but also beyond the time of the events, as almost four decades have passed since the dictatorship ended. Such a pushed-forward view is fundamental for understanding how collective trauma is constructed in collective memory and how it models identities; a topic which, in the Argentinian case, has been long ignored in the social sciences literature. This research's impact extends to the societal level, as Argentines' voices and stories about their country's history are often silent in Europe, where the events concerning the *desaparecidos* are not commonly discussed. Additionally, this study includes the diaspora's opinions on Argentina's approach to transitional justice, such as about the National Commission on the Disappeared (CONADEP) and the trials of the military junta, which are essential for estimating the long-term effects of such provisions on the targeted population.

The content of this research can be separated into two sections: a historic-theoretical section and a qualitative study. Firstly, the paper will present the historical context of the dictatorship and its developments, followed by the theoretical account that I used as a departure point for my qualitative study. The subsequent section outlines the narratives regarding the political oppression that emerged in Argentina after 1983, based on the works of Antonius C.G.M. Robben (2018), which are used as an analytical base for the qualitative study. A description of the qualitative study follows, and its methodology and analysis are set forth, together with a final discussion and conclusion.

1.1 A Brief Synopsis of Argentinian Political History

To understand the socio-cultural context in Argentina, it is fundamental to step back and make sense of the historical succession of political struggles and of the social issues that derived from them. Argentina gained independence from colonial rule in 1810, as the product of a brutal anti-colonial war. The process of independence, however, did not result in a peaceful and well-adjusted society. Instead, civil wars between the central government in Buenos Aires and peripheral rebels continued for 40 years (Robben, 2018, p. 6). Argentina finally found prosperity in the late twentieth century, but Buenos Aires soon became a center of exploitation for migrant workers who left Europe to escape poverty after the First World War. The resulting civil discontent turned into strikes and riots that were violently repressed; 34 striking workers were killed in 1919 by

law-enforcement units during what is known as *La Semana Trágica*, which translates to "The Tragic Week" (Romero and Brennan, 2013, p. 32). From this moment onwards, political oppression as a standard modus operandi never left Argentinian society and continued even after the end of the last military dictatorship in 1983. Three major periods of military authoritarianism characterize Argentinian history. The first military regime took over in 1930 and ended in 1945, when Juan Domingo Perón was elected president. Perón presented himself as the savior of the working class, but soon became a more authoritarian and controversial figure. Indeed, the first disappearances ever documented in Argentina happened during his rule: in 1946, five detained political opponents were distributed between different police stations, and their custody was publicly denied by the officers (Robben, 2018, p. 7). Such methods would become the norm in the 1970s, when under the generals Videla, Viola, and Galtieri, 30 thousand political opponents met the same fate and became the often-called 'ghostly figures' now known as *the desaparecidos*. Perón's first term was halted by a military coup in September 1955, which followed the bombing of Plaza De Mayo in June 1955, where the Navy caused hundreds of civilian casualties (Portugheis, 2015). Perón consequently left the country in exile. During the second military regime, rebellious Peronist groups composed the political opposition and tried organizing in guerrillas, which were immediately met with oppression. Moreover, the Peronist groups did not constitute a unitary opposition, but instead were divided into the left-wing revolutionary Peronists – affiliated with the newborn Marxist People's Revolutionary Army (ERP) – and the right-wing orthodox Peronists. The prominent street protests and wide-spread discontent succeeded in pushing the military junta to organize elections in 1973, which resulted in victory for the Peronist Party, and Héctor Cámpora became president (Romero and Brennan, 2013, p. 125). However, the new political situation did not settle, and the internal conflict grew more violent by the left-wing Peronist guerrilla *Movimiento Peronista Montonero*, also known as the Montoneros, and other right-wing armed groups that were supported by labor unions and some military officers. Perón came back from his exile in October 1973 and ruled until his death in July 1974. His wife, Isabel Martinez de Perón, took power but her administration was characterized by a domestic turmoil caused by rising violence between the rebel groups (Romero and Brennan, 2013, p. 204). The situation escalated until, on March 24 1976, the junta conducted the third coup, and general Videla became the head of state, followed by generals Viola and Galtieri, who ruled until 1983. The regime justified its methods in coming to power by citing the previous political unrest of the country and established the following objectives for the dictatorship: remove the varied revolutionary movement, bring economic prosperity, and renovate society entirely through the "Process of National Reorganization" (*Proceso de Reorganización Nacional*), which aimed at instituting democracy and protecting Argentinian culture and religious Christian values (Robben, 2005, pp. 171-189). It is relevant to this research to note how the junta pinpointed and identified a range of ideological and cultural norms as representative of "Argentinianness". Moreover, the inclusion of religious morals and principles in the military's self-justification shows how religion was instrumentalized, becoming a strong and legitimizing force for political action.

This section provided the background needed to understand the socio-political context in which the last military dictatorship was situated in Argentina. Moreover, it introduced the main actors involved in the struggles during and after the dictatorship. The next section outlines the mechanisms and tools of state repression employed by the junta between 1976 and 1983.

1.2 What happened to the *desaparecidos*?

As the word *desaparecido* will be widely used in this study, it is fundamental to make clear what the term indicates, and the experiential meaning it entails. Most *desaparecidos* were active political opponents to the junta, but many were also civilians who had little to no ties with opposition groups (Robben, 2018).

In 1976, shortly before the third coup took place, the Vice-Admiral Luis Maria Mendía told the following words to his officers:

> "We shall act with civilian clothing, in quick operations, intense interrogations, practices of torture and physical elimination by means of operations in aircraft from which, during the flight, the living and the narcotized bodies of the victims will be thrown into the air, thus giving them 'a Christian death'" (Feierstein, 2006, p. 152).

Mendía describes a technique widely used by the junta during the 1976-83 dictatorship called "death flights". Navy Captain Scilingo, a controversial figure that, in the early 2000s, gave important testimonies for the reconstruction of the junta's means of extermination and publicly referred to "death flights" as some of junta's most common means (Robben, 2018, pp. 110-120). The crude image of the death flights is only one way the military treated political opposition. This section outlines what is currently known about the events that characterized the detention of the *desaparecidos*.

Robben (2018) highlighted that since the last dictatorship started, the commanders referred to political antagonists as a "cancer of society" and a "disease" that was infesting Argentina (p. 44). This discourse, as Robben points out through quoting the works of Sontag (1990), is a strong encouragement to violence and a justification of extreme means for solving such a pressing

problem. Moreover, Robben continues, the exterminating goal of the military was prominently ideological, as it implied the destruction of political beliefs and agency. Robben (2018) claims the "battle had to be won on the terrain of the human psyche", and victory would be secured when revolutionaries would be physically eliminated and the survivors would abandon their political ideals (p. 44).

A brief description of how the standard abductions of civilians took place follows. Officers, as Mendía's speech mentioned, disguised as civilians would simply ring the doorbell of the suspected citizens' houses, and would find their way in (Robben, 2005, pp. 223-228). The officers would then find the suspects, search them and their houses, and proceed to torture them, often in front of their family members, who were also the targets of violence. The victims were then blindfolded and brought to a secret detention and torture center. Here, convicts were depersonalized: undressed, they were given a number, and tied to a bed frame. Thrashings, humiliation, and sexual abuse were customary. Torture was used to collect information about the guerrilla organizations the victim belonged to and was carried out for hours, even if it did not result in new information. Captives were also dehumanized, often called animals and instructed to act like them. Traumatization was seen as even more powerful than assassination, as it allowed the regime to destroy a person's mental capabilities and sense of self, and to dismantle their political agency (Robben, 2005, p. 211). Pregnant women were often subjugated to forced abortion because of the physical abuses. Another common practice was the torture of infants, often in front of their parents. Many times, infants would be taken from their parents and secretly given in adoption to families of the junta. These acts were clear displays of the military's power over the citizens' treasures, especially the emotional ones. Robben suggests that the appropriation of detainees' babies should be understood as the seizure of a "war booty" (Robben, 2005, p .296). He also argues that this strategy was implemented by the military to stop the adversary's "social, emotional, and ideological reproduction" (Robben, 2018, p. 126).

The most famous example of these detention and torture centers is the Escuela Superior de Mecánica de la Armada, which detained a total of around four thousand disappeared (ESMA, in English: Navy Mechanics School). Here, the torture treatments produced either "harmless citizens", who were then released into society, or "recoverable patients", who were then sent to the ESMA rehabilitation program (Davis, 2013, p. 3). A similar rehabilitation center was based in the Campo de Mayo army base. Robben (2005) reports that the program's objective was to "desocialize" the captives from their revolutionary worldview and "resocialize" them into the regime's ideals (p. 255). When entering the program, inmates were given clothing and larger cells, and depending on their behavior, some were assigned little jobs, such as photographing the new detainees. Once they had been effectively desocialized, for example by having no emotional reaction to the torture of other convicts, they passed on to the "unshackled" level (p. 255). The latter allowed them access to better food and restrooms, and seldom even permission to go home for the night. Seventy percent of the detainees in the ESMA program were released after numerous psychological tests that would prove that the resocialization took place.

It is important to point out that the political repression, despite the military's claims, which are outlined in the next paragraph, was systematic and premeditated. There were an estimated 520 detention centers throughout the whole country (Instituto Espacio para la Memoria, 2007), although they all slightly differed in modalities and treatment of their inmates. Moreover, Montero (2019) argues the detention and torture centers were only the operational nodes of a larger organizational structure. The latter also included semi-legal sites that functioned alternatively as either "whitening spaces" (Montero, 2019, p. 72) (whitening – *blanqueamiento* – is the process of bringing a person from a clandestine and illegal place of confinement to another one where there was a record of their detention, to deflect recording traces), detention centers where no torture was involved, or other institutions that aimed at legalizing detentions, such as the Penitentiary Unit 4 of Villa Floresta.

The practices summarized were widely ignored and denied for a long period, even as the dictatorship ended in 1983 (Robben, 2018). Nevertheless, as the sociopolitical situation in Argentina changed over the years, the events of the dictatorship were slowly revealed and gained attention in public discourse. The process that caused this awareness to grow was convoluted and controversial, since the emerging narratives often contradicted the government's. Moreover, the narratives varied in their assignments of the roles of "good/righteous" and "evil/morally wrong" to the actors involved. These narratives will soon be described; however, it is fundamental to first present a theoretical understanding of how this part of Argentinian history was scholarly interpreted in order to show the relevance of this historical period.

2 Theoretical Framework

2.1 Collective Memory

In his extensive work on the country's history and society, *Argentina Betrayed*, Robben (2018) argues that when the dictatorship ended in 1983, Argentina emerged as a socio-culturally traumatized society. Social trauma, he explains, implies the breaking of social bonds, such as families having been separated, which destroys the fundamental sense of collectivity and dam-

ages the community's harmony in terms of social and political trust (Robben, 2018, p .34). Robben's analysis centers on feelings of trust and distrust, arguing that fear operates through the latter, making trauma permeate the texture of society. Argentines not only stopped trusting each other, as they feared being reported as political opponents even if they were innocent, but they also lost trust in their authorities who for decades did not recognize and downplayed their sufferings, even pardoning the perpetrators of their agonies. Robben argues that Argentina also faced cultural trauma: the group consciousness, memories, and future identities were forever marked by the violent events (Robbens, 2018, p. 35). Social and cultural trauma, he reasons, continuously interact and interplay with each other, therefore giving rise to socio-cultural trauma (p. 40).

Likewise, Edelman et al. (2003) analyzed the psychosocial effects of Argentina's mass trauma and of the discourses that derived from it. They argue that the officers' public denial of the events and the lack of information about what happened to the disappeared furthered frustration among the population. The authorities' contradictory behavior peaked when their denial was paired with the insistence that the relatives of the *desaparecidos* should consider them dead. Yet, the families' bereavement was hampered by the absence of dead bodies or any physical evidence of their loss, and trauma-processing kept being disturbed and deferred because of the convoluted and de-legitimizing narratives that were employed by the regime (Edelman et al., 2003, p. 144). These dynamics resulted in "catastrophic" socio-political consequences, which

> "have already been repeated in succeeding generations as frustrated expectations of social justice gave away the widespread personal experiences of skepticism, alienation, the loss of collective social values, and disillusionment with the authorities" (p. 145).

Stockwell (2011) argued that the persisting ruptures in Argentinian society are to a large extent due to the public sharing of traumatic experiences by women whose partners, children, or grandchildren were abducted and/or killed. She argues that emotional sharing strengthens societal polarization, which has been long characterized by antagonism between the *victims* and *perpetrators*. Emotional sharing reinforces the traumatic character of the events, enhancing the hostility toward those who are perceived as guilty of the atrocities. Moreover, the author emphasizes the heavy presence that *desaparecidos* still occupy within the society, like ghosts that haunt the population, making the act of remembering tied to specific ideologies and political values (Stockwell, 2011). The different memorial cultures have no space to evolve and merge when remembering becomes a political act in an increasingly polarized environment. Nevertheless, Stockwell also sees the potential of emotional sharing as a bridge between the two sides, as it blurs the borders between the self and others.

Crenzel (2020) also uses the metaphor of 'haunting ghosts' in relation to the *desaparecidos* and Argentinian society. He argues that it is the institutionalization of a "narrative of the horror" (for instance, through the establishment of the CONADEP truth commission, the trial of the military juntas, and the memory sites) that gave "citizenship status to the ghostly figure of the disappeared" (p. 263). He claims that the transitional justice [1] approaches were significantly inadequate, as they assimilated these ghosts into the victim society to an extreme level. Contrary to Robben's perspective, Crenzel argues that Argentinian society perceives itself as unconnected to the violence, as all the blame is now projected onto the former regime and a sense of perpetuated victimhood remains together with the ghostly figures of the *desaparecidos*, which hampers national advancement and social change.

In summary, the literature tends to associate the events of the *desaparecidos* with the concepts of socio-cultural and mass trauma, which are tightly related to collective memory and narrative. It is important to see how trauma took such a central role in the interpretation of such events and their consequences. Therefore, a brief account on the recent prominence of the concept of trauma in the social sciences follows, together with an explanation of the relationship between trauma and collective memory, which I took as the starting point for this study and the qualitative part of this research.

2.2 Shared Trauma

Fassin and Rechtman's *The Empire of Trauma* (2009) is a comprehensive anthropological analysis of how trauma gained increasing attention in the world of the social sciences, not only in the field of psychiatry but also in broader social contexts. The authors attribute the beginning of the modern discourse around trauma to the late 20^{th} century. In 1980, the American Psychiatric Association published the third edition of the Diagnostic and Statistical Manual of Mental Disorders (DSM-III), where for the first time the clinical condition of *Post-Traumatic Stress Disorder* (PTSD) was introduced in the history of the DSM. Here, the first classifying criterion was for the patient to have experienced a major stressful occurrence, which would later induce symptoms of distress. According to the Manual, no weak personality trait nor predisposition was needed – anyone could be affected by PTSD – which is now understood as the physiological reaction of any "ordinary person" who confronted an "extraordinary event" (Fassin & Rechtman, 2009, p. 87). The publication of the DSM-III triggered

[1] The UN Security Council (2004) defines transitional justice as: "the full range of processes and mechanisms associated with a society's attempts to come to terms with a legacy of large-scale past abuses, in order to ensure accountability, serve justice and achieve reconciliation".

many controversies and critiques, initiating a "social reform" in the US psychiatric world (p. 85) as the figure of the "traumatized victim" was given unique importance. This created a new discourse that views trauma as the most genuine authenticator and legitimizer of suffering. Concurrently, the frequent association between PTSD and veterans of the Vietnam war and the emerging discoveries about the genocide of Jews in Europe made the concept of trauma increasingly connected to collectives and not only to individuals. These discourses made "trauma the universal language of new politics of the intolerable" (Fassin & Rechtman, 2009, p. 97), giving violence major social significance as a cause of long-term sufferings at the micro- and meso-level. Whilst individuals remember their traumatic experiences through personal memories, *collective memory* became recognized as the mechanism of remembrance and processing for entire groups and societies. Fassin and Rechtman (2009) define collective memory as a "traumatic relationship with the past in which the group identifies itself as a victim through its recognition of a shared experience of violence" (p. 16). Therefore, the events that are interpreted as collective traumas have the power to construct a social group's identity and self-perception. Additionally, the authors argue, subjectivity and collectivity are fundamentally complementary, as within the collectiveness of the shared trauma individual suffering is still central because it acts as a brick in the wall of collective memory and bears concrete witness to the traumatic event (p. 18).

In his writing about how sociological theory has approached the concept of memory, Jedlowski (2001) uses Halbwach's concept of "social frameworks" (1975) – which describe the socially-constructed structures that give meaning and significance to memories – and adds that they are spread and perpetuated through language and discourse, which he calls "narrative practices" (Jedlowski, 2001, p. 34). The institutionalization of such memories – through, for instance, commemorative practices – gives space and legitimization to a group's self-expression, which contributes to creating collective identity (more specifically, he talks about *national identity*) both symbolically and cognitively. Following the same logic, Kenvers (2016) argues that individual memories are formed according to the conceptual makeups and habits of the person's group of belonging, which makes them tightly related to the narrative adopted by the group itself. Therefore, individual and collective memories are interrelated and co-constitutive.

A practical example of the interactions between collective memory, trauma, and narrative is given by Kaya's (2018) study on how the Armenian diaspora in Canada remembers the Armenian Genocide of 1915-16. Although she observed different means of commemoration in her analysis, she points out that they all share the same themes and narrative. These discourses were reinforced by the creation of committees and organized manifestations, which strengthened and materialized transnational memories. What comes out of these processes is the "impression of a singular experience of Armenianness based on the collective memory of the Armenian Genocide" (Kaya, 2018, p. 415). A shared past, although more than a century old, is thus preserved through collective memory and it cements the group's identity.

Building on this conceptual framework, this research aims at discovering whether the Argentinian diaspora in Europe holds on to their traumatic past in the same way, what the prevalent narratives are that frame the shared experiences, and what role they play in identity creation. The following paragraphs provide an account on the six main narratives around the events of the *desaparecidos* in Argentinian society, as detected and identified by the ethnographical work of Antonius Robben (2018) on Argentina's socio-cultural trauma deriving from its internal violence in the 20^{th} century. Secrecy, denial, war discourse, state terrorism, and genocide will be brought together to argue how they produce and shape what Fassin and Rechtman (2009) call "collective trauma". These discursive frames will be individually explained and used as a departure point to explore how the Argentinian diaspora living in Europe adopts or relates to them.

3 Narrating the *desaparecidos*

3.1 Secrecy: an inaccessible history

The phenomena of the disappearances justly caused panic and fear for the relatives of the victims, who engaged in desperate searches for their loved ones for years, and often decades. Unfortunately, many of these searches never resulted in clear answers to the relatives' questions: some found closure thanks to the forensic exhumations of anonymous graves – which took place from the mid 1980s and until the early 2000s (Robben, 2018, pp. 137-123) – whilst others had to accept their unresolved losses, hovering between the acknowledgment of death and the hope for a survived existence elsewhere. The first approach that the military junta took in relation to their repressive techniques was secrecy. Secrecy was the discourse that predominantly framed the phenomenon of the *desaparecidos* throughout the dictatorship, until the reports of the CONADEP truth commission came out in 1984, which provided proof of the illicit political repression (Robben, 2018, p. 113). Secrecy was bluntly kept in relation to the relatives of the *desaparecidos*, as they turned to the police, military, and administrative offices to report and inquire about the missing persons and ask for their news. No information was ever leaked to the civilians, who regularly had to face the fake ignorance of officers, the false promises of future efforts to find their loved ones, and the ignored requests of *habeas corpus* - requests to

see the detainee's conditions to prevent unlawful detention (p. 111). The junta's secrecy, Robben argues, was strategic for three main reasons: it withheld power from the population, it reinforced military unity, and it protected the junta from accountability (p. 113). Secrecy, however, failed as soon as signs of what happened became evident, and denial gradually took over the military's narrative.

3.2 Complete Denial: negation as the answer

The way the military junta employed the rhetorical tool of denial changed over time and adjusted itself to the changing climate that surrounded Argentinian society in the years preceding and following the end of the dictatorship. Cohen (2001) identified three phases of how denial is usually handled in official discourse: "*literal* (nothing happened); *interpretative* (what happened is really something else); and *implicatory* (what happened is justified)" (p. 103). I will call the first rhetorical approach of the junta "complete denial", as for an initial period, the junta did not even acknowledge the fact that Argentinian citizens were disappearing, which coincides with Cohen's *literal* phase. Cohen's two other phases later produced the "dirty war" and the "two demons theory" discursive frames, which will be explored in upcoming paragraphs. Complete denial was implemented when more and more complaints emerged from Argentinian society, and people required an explanation for the growing evidence that something illegitimate was taking place. Initially, the officers blamed the guerrilla groups and the right-wing death squads for the abductions and the violence, arguing that those methods were not within the military's operational schemes (Robben, 2005, pp. 264-269). A clear example of complete denial happened when the Inter-American Commission on Human Rights (IACHR) visited the country and conducted investigations in 1979: they reported that between 1976 and 1979, thousands of people went missing in the Argentinian territory (Robben, 2018, p. 116). The junta's official response indicated that the reported disappearances were merely a strategy employed by revolutionary groups, purposefully created by the terrorist insurgents that went into hiding, who were later reported missing or dead by their families (Government of Argentina, 1980, p. 54).

Robben (2018) argues that denial is particularly problematic as it "relegates experiences to the realm of fantasy" (p. 54). Instead, people who faced such intense and traumatizing events, both the victims and the perpetrators, need universal recognition of the reality of their lived experiences, and require them to be integrated into individual and collective memory. Moreover, Robben puts forward a noteworthy argument that interprets the long-lasting denial by the military as a mechanism of the overarching Argentinian socio-cultural trauma (p. 53). Denying, and therefore not recognizing the brutality that one has committed, is a natural human response that attempts to downplays the consequences of one's own actions. Therefore, according to Robben, denial was one of the symptoms of the socio-cultural trauma reigning in Argentina after 1983, and not just a rational strategy employed by the junta.

3.3 *Guerra Sucia*: just a "dirty war"?

It is impossible to identify one single cause for the ending of the last Argentinian military dictatorship. Nevertheless, it is recognized that the culmination of the following factors played a decisive role: inefficient economic policies, rising discontent and public demonstrations due to the growing awareness around the violent political repression, and the military defeat of the Falkland Islands against the United Kingdom (Romero & Brennan, 2013). This cascade resulted in the establishment of a transitory government in 1982, with General Bignone as the head of state. In April 1983, Bignone ordered and executed the destruction of all documentation linked to the oppression, which later complicated the investigative process taken over by the CONADEP truth commission. During the Bignone administration, the *dirty war* rhetoric gained more and more public space. This narrative sees the events of the dictatorship as belonging to a *guerra sucia* (Spanish for dirty war), which the junta employed to liberate the country from imminent threats to the national wellbeing and western Christian values embodied by the revolutionary opponent groups, often referred to as "subversive" (Robben, 2018, p. 63). The discourse of *guerra sucia* still rejects the claims that systemic repression was employed by the military, but it recognizes that during the dictatorship the confrontation between military forces and guerrilla groups entailed violence and abuse. Excessive violence and abuses, however, are inevitable during war, and violence knows no limit in such times of crises. The *dirty war* narrative could be described as Cohen's second, *interpretative*, phase of denial, because it reasons that the object of speculation is not really violent state repression, but rather only a combination of natural consequences of what a war entails.

The retired General Osiris Villegas argued that any means are justified in war, as long as they weaken the enemy while minimizing one's own losses. He went on to draw a parallel between Argentina's case and the destruction caused by the atomic bombs dropped in Japan during the Second World War (Villegas, 1990, p. 67). Another intervention worth mentioning are the words of General Díaz Bessone during an interview with Robbens in 1989: "We haven't violated the law of God. For me, this is a just war because we weren't the ones who attacked" (Robben, 2018, p. 63). The National Commission on the Disappeared (CONADEP) was established in December 1983 with the purpose of investigating how the junta operated in the years of the dictatorship. The

CONADEP found substantial evidence through visiting former detention and torture centers, talking to survivors of the repression and relatives of the *desaparecidos*. In September 1984, the *Nunca Más* ("Never Again") report was published, representing the first methodical account of the repressive military operations openly accessible to Argentines (Robben, 2018, p. 64). The military, which by then was not in charge anymore, reacted by reiterating the *dirty war* discourse by arguing that the junta had the "constitutional right and obligation to defend Argentina's sovereignty" (Robben, 2018, p. 117), and by denying that torture and premeditated repressive means were ever employed.

In sum, the *dirty war* argument appealed to the ruthlessness of war, to the tragic nature of the historical moment, and to the constitutional duties of the junta as the deliverer of order and prosperity. This narrative developed complementary to the *two demons theory*, as both avoid the full acknowledgments of the misconducts of the junta. However, the following analysis separates the two because they differ in the distribution of blame between the two parties involved.

3.4 The Two Demons Theory: equal adversaries

President Raúl Alfonsín was the first elected president after the dictatorship, coming to power in December 1983. The establishment of the CONADEP truth commission was one of the first actions after his election. Despite the important step towards national recovery that the founding of CONADEP represented, the narrative the Alfonsin administration pushed was still problematic in the way it portrayed the story around the *desaparecidos*. The *two demons theory*, arguably the most prevalent during those years, seems to correspond with Cohen's (2001) *implicatory* phase of denial, as it justifies the junta's actions. The *two demons* narrative claims that during the years of the dictatorship, the two parties involved (military junta and subversive groups) in the ongoing ruthless war (hence the direct connection with the *dirty war* narrative) were equal in means, cruelty, and fault. While the *dirty war* narrative leaves space for the recognition of abuses from the military's side, blaming the factors mentioned above, the *two demons theory* does not, and suggests that both sides were equally driven by a moral compass.

When in 1984 the CONADEP report was published, one of its most important conclusions was that many of the disappeared were dead, but their bodies had not been found yet because they were destroyed or dismantled as part of the disappearance planned policy (CONADEP, 1986, p. 233). The searching relatives of the victims found this statement harrowing, as it denied the possibility that any of the missing people could still be alive, confirming that they were "dead yet unaccounted for" (Robben, 2018, p. 64). Moreover, the prologue of the report stated that the military reacted to the opposition's terrorist crimes with a stronger and crueler terrorism than the one they were fighting against (CONADEP, 1984, p. 1). Although this claim recognized the imbalance of means that were employed, it still stressed that the escalation of violence was caused by both terrorist factions. Robben (2018) reasons that the 1984 CONADEP report supported the *two demons theory*, as it reinforced the narrative of two evils fighting against each other, instead of victimizing one side and condemning the other (p. 65). On a judicial level, Alfonsín's administration initiated a succession of trials and amnesty laws, in the name of national reconciliation, that fueled the *two demons* narrative. In 1985, the trials of nine commanders started, which was seen to promote accountability and punitive justice, and more prosecutions of the military immediately followed. On the other hand, the prosecution of seven guerrilla commanders also initiated an enactment which reinforced the *two demons theory* (Robben, 2018, p. 65). Furthermore, Alfonsín's recovery-oriented approach to national reconciliation interpreted the trials, especially those against the military, as a polarizing force that created discontent in the military bodies and antagonized the armed forces for the population. In December 1986, the Congress approved the *Ley de Punto Final* (Full Stop Law) which enforced a sixty-day limitation period over more indictments, meaning no one else could be declared guilty during that time. In June 1987, the *Ley de Obediencia Debida* (Due Obedience Law) was implemented: only high-ranking officers could be prosecuted and only the crimes of theft, rape, and kidnapping of infants could be indicted (Robben, 2018, p. 69). This decision freed many of the officers who were already indicted and caused public discontent, especially among human rights organizations and searching relatives. The president that followed Alfonsín, Carlos Saul Menem, whose term lasted from 1989 to 1999, adopted the same recovery-oriented policies and pardoned hundreds of members of the military under prosecution, including the Generals Viola and Videla, together with 277 members of the resistance while arguing that it would result in "national pacification" (Robben, 2018, pp. 69-70). Robben explains the problematic aspect of the amnesty laws by arguing "the assassinated-disappeared had become negotiable losses by absolving the material authors and sacrificing society's right to redress, to benefit democratic consolidation and social peace" (p. 213). The *two demons theory* was short-lived as Argentines became increasingly aware of the wrongdoings of the junta and of the asymmetrical nature of the 1976-1983 conflict.

3.5 State Terrorism: blaming the military

The finding and exhumation of mass graves and the growing power gained by human rights groups, such as the Mothers and Grandmothers of Plaza de Mayo, played a crucial role in the recognition of the traumatic experiences of survivors and families of the *desaparecidos*. They had an important impact on the expansion of the narrative that framed the junta's crimes as state terrorism. At the beginning of the twenty-first century, state terrorism became the prevalent discursive frame, and it found institutional recognition with the administration of the fourth Argentinian president after the end of the dictatorship: Néstor Kirchner, whose term took place in the years 2003-2007. Kirchner shifted national reconciliation policies from a general recovery-oriented approach to loss-oriented accountability and recovery-oriented memorialization (Robben, 2018, p. 217). *Ley de Punto Final* and the *Ley de Obediencia Debida* were annulled in 2005, after which posthumous human rights trials were conducted. Between 2006 and 2016, the number of accusations reached 2,436 (Robben, 2018, p. 168). Meanwhile, the former guerrilla members remained legally absolved. The discursive frame of state terrorism gave validation to the lived experiences of the Argentine people and was reinforced by the growing body of evidence that was fueled over the years. In 2006, Eduardo Luis Duhalde, the secretary of human rights, wrote a new preface for the CONADEP report which overruled the previous *two demons theory* and instead stressed that the junta operated through means of state terrorism (Duhalde, 2017). Robben pointed out that the narrative of state terrorism was already employed during the dictatorship by Argentines in exile living abroad, who already recognized that the junta employed more violence than the rebel groups (Robben, 2018, pp. 70-71).

3.6 Genocide: a new frame

Fassin and Rechtman (2009) argued that the Holocaust in the modern era became the "paradigm" of collective trauma (p. 18). Since it became interpreted as the most extreme representation of violence, it exemplified the "reference point" for any other episode of mass violence. Over the years, a transnational discourse around victimhood became increasingly prominent, and from 2005 onwards the term "genocide" was introduced in Argentina to describe the 1976-83 events. The genocide narrative was easily welcomed by survivors and relatives of the victims, human rights groups, and social scientists. In 2007, the *Revista de Estudios sobre Genocidio* (Journal of Genocide Studies) was funded, which fueled the parallelism with the Argentinian political repression and the persecution of the Jews in Europe in the academic environment (Robben, 2018, p. 77). Many authors who researched about the dictatorship classified its means as genocide (Bianchi, 2009; Caviglia, 2006; Cieza, 2009,). Important Argentinian scholars are included in this list, such as the political scientist Pilar Calveiro (1998), and the sociologist Daniel Feierstein (2006; 2008), whose analysis stressed the systematic nature of the junta's efforts for the annihilation of its political opposition and the forced imposition of their plan for national configuration. Nevertheless, the narrative of genocide also raised important controversies. Firstly, the same years were also characterized by the rising of voices from the new generation that shed light on the violence inflicted by the guerilla groups on the military, victimizing members of the junta as well (Robben, 2018, p. 79). Secondly, authors such as Vezzetti (2003) pointed out that there were discrepancies between the events under the junta dictatorship and what the term "Holocaust" entails. Vezzetti argued that the victim group in Argentina was targeted because of its political stance and its suspected or real actions, instead of some intrinsic characteristic it belonged to, such as religion or ethnicity, (Vezzetti, 2003, pp. 153-164). Moreover, Robben argues that the genocide discursive frame transfers guilt from specific individuals to a general collective, as the term "genocide" implies the moral collusion of the entire society without lifting the burden of responsibility from anyone (Robben, 2012, pp. 311-312). The shared responsibility hinders national reconciliation, and risks perpetuating the socio-cultural trauma that the events caused within Argentinian society. Nevertheless, historically speaking, the genocide discursive frame is the most recent and probably the one that answers the need for recognition of victimhood that the Argentinian people have longed for for decades. Moreover, it reflects what Fassin and Rechtman (2009) would define as modern society's growing gripped preoccupation with collective trauma and victimization.

Considering the difficult process of coping with this trauma, it is important to look at personal perspectives to understand how these overarching mechanisms are reflected at the individual level. Argentina's diaspora in Europe is a relevant object of study as the physical distance from their homeland complicates the dynamic between collective memory, narratives, and individual remembering.

4 Qualitative Research

4.1 Methodology

For the qualitative part of this research project, I conducted semi-structured interviews with five Argentinians currently living in Europe, with the exception of one, who is now staying in Bali, Indonesia, but lived for nine years in Norway and two years in the Netherlands. The small sample was due to the limited time I could dedicate to this research, and to a lack of responsiveness among the people I tried to contact. The selection crite-

ria were: 1) participants had to be Argentines that lived in Europe, 2) they had to be older than 40 years old, to ensure exposure to the different narratives that developed in the country in the past decades.

The other participants currently live in the Netherlands or Italy. Three interviewees out of five were recruited through personal contacts, whilst two of them contacted me after I reached out for volunteers to be interviewed in a Facebook group called "Argentines in the Netherlands". I posted an announcement in the Facebook group mentioning that I was looking for Argenintes to interview and that the topic of my research involved the *desaparecidos*. All interviews but one were conducted online because of the physical distance between us or the impossibility to meet otherwise. All participants were informed about my research question at the beginning of their interviews. As anonymity and respect for privacy of the data was assured to all participants, I will here report their words and arguments using pseudonyms. All interviews lasted between 60 and 90 minutes. Moreover, given the sensitive nature of the topic of research, the participants were warned about possible triggers and asked to answer only if comfortable. The questions of my interviews inquired the participants' first-person experiences with the political oppression, the exposure to the different narratives explored in the first half of the research, and what role their country's history played on their lifestyle and social bonds once they moved abroad. The interviews were carried out both in English and in Italian. The participants' pseudonyms and ages are: Pedro (43); Maria (46); Juan (40); Valentina (52), and Carlos (69). Maria and Valentina are currently living in Italy, and their interviews were conducted in Italian. I did not translate the interviews' transcriptions for the data analysis, as I could easily navigate the two languages simultaneously, however, when I had to report their words in my findings, I translated them while attempting to preserve the participants' jargon and figures of speech.

I had heard about the *desaparecidos* when I read the book "Argentina Betrayed" by Robben (2018), the main reference point of this research. Moreover, the novel "Mothers and Shadows" by Marta Traba (1986) made me dive deep into the history and personal chronicles of the *desaparecidos*. As a 21-year-old Italian student living in the Netherlands who never experienced such strong violent occurrences around her, I was aware of my privileged position compared to people who instead had to deal with constant political turmoil. Moreover, I feared that the age difference between me and the interviewees would not make me a credible researcher. Nevertheless, my worries were dismantled once I started conducting the interviews: some of the interviewees came from Italian descent and joked about the similarities between our cultures, and our age difference made them sympathize with me as a young student, instead of creating distance between us. Through the data analysis I classified the interviews' results into five recurrent and significant themes, which will be outlined in the following paragraphs.

An important limitation of this study is my lack of a direct experience in Argentina, as I have never been to the country. This shortcoming makes my background information about Argentinian history and its societal impacts incomplete, because I solely rely on scholarly sources and other informative means, such as novels and documentaries. Moreover, it is worth mentioning that this study does not aim at drawing parallels between the experience of the Argentinian diaspora with the Argentinian population.

4.2 First exposure to the *desaparecidos*

Before starting the data analysis, it is fundamental to point out that all the interviewees seemed very familiar with the case of the *desaparecidos*: who they were and what happened to them. During the interviews, I had the feeling that they felt the duty of explaining to me that it was part of Argentinian history: it did not matter how much I read or researched about it; they knew more about it just because they are Argentines, even if they did not remember anything or were not born yet during those years. This dynamic played out well for the research because I was exactly interested in the way they would tell the stories they know: under which frame and from which perspective?

Given the secrecy and complete denial that surrounded the events of the last military dictatorship, it was interesting to learn how the Argentines I interviewed got acquainted with the reality of what had happened. While analyzing the data, I discovered how different the experience was for each person, which seemed to be largely connected to their exposure to the events, and which varied depending on their age, their families' political orientation, and education environments. Carlos, born in 1952, is the only one who was old enough to have first-person experiences of the dictatorship. He says that the first time he heard and realized what the junta was doing to the *desaparecidos* was in 1978-79, when an airplane directed towards the Aeroparque airport in Buenos Aires tragically fell into the waters of Rio de La Plata. I could not find any information about this accident, but one that corresponds to Carlos' description happened in May 1981, where a Bac-500 plane belonging to the Austral airline ended up in the river, causing thirty casualties (Clarin, 2001). Carlos said that when the authorities went to collect the victims of the accident, they also found many more corpses in the river: "But the problem was, they started[ed] to take people that were not in the plane. And those were the people that the military... They usually went in flight and jump the people in the water". He was referring to the death flights. Carlos said that until that moment, most people assumed that the *desaparecidos* were hidden or imprisoned in some far-away city; instead, that event showed the brutality of what was happening be-

hind the scenes. The other interviewees of my research were too young during the dictatorship to have had first-person experiences with the events. All of them had said that it was mainly through the stories of friends or neighbors that they slowly managed to compose the puzzle of what happened during that time. Family was the most influential source of information for all of them, but in an indirect and subtle way. As they were of a young age, they were never given a clear, well-reasoned and explained narration of how their political oppression worked and why. Pedro is the interviewee with the most significant family involvement in the dictatorship, as his father was part of the Partido Obrero (Workers' Party), and a member of the logistics of the left-wing Peronist guerrilla group Montoneros for a limited period of time. However, Pedro said all the stories he knows were not told to him by his father, because he died at a relatively young age, but by his father's best friend, who was by his side during his political involvement. For both Maria and Valentina, their families would mention and talk about the oppression when it came to safety issues, recommendations on how to behave, the environments to attend and those to avoid. The same happened to Juan, who recalled:

> "I do remember that my parents [...] it was not only my parents, also other parents will say things like, well, you should never walk in the street with your agenda with all the phone numbers, you know, those things were in the back of the minds. [...] When somebody calls on the phone, never say who you are, always ask. Who are they? Who are they? Who they are not? Don't give away who you are. [...] But there was this fear that, or the idea that you should always go out with your ID. That was, let's say, in the 80s. I was very young."
>
> - Juan (46, Argentine living in Amsterdam)

Through this description, it is observable that there was a culture of fear, suspicion, and self-preservation, which was very present in the parents-children pedagogical dynamic. Moreover, during the interviews I also explored the role that education played in informing the participants about this fundamental aspect of Argentinian history. My results indicate that Maria, Valentina and Pedro were never exposed to such topics in school. They said their school curriculum was more centered around the nineteenth century and the country's colonial past. Maria attended a school managed by nuns, where the topic of the dictatorship was a "taboo for many many years," but as she went to university, and studied in the faculty of Sociology, everything changed. Here, the events of *desaparecidos* and the reigning social injustice during the dictatorship were central objects of study. She also got in touch with fellow students who had had direct experiences with the oppression: "it was very likely to find people whose mother was a desaparecida or went to jail for a while". Maria made it very clear that her experience in university was extremely enlightening for her, as she dived into the history of the *desaparecidos* at an academic and personal level; before that, she felt quite ignorant and detached from it. On the other hand, Juan reported a very different experience in the public education system. He is the youngest of the interviewees, and by the time he started secondary school, recovery-oriented memorialization policies had started (Robben, 2018, p.217). Juan said that his school was public and involved in politics, so he had plenty of chances to get acquainted with the events of the dictatorship, especially through protests and commemorations. He described March 24 as the most important remembrance day:

> "I think the most important activity for secondary school students, even now, even after so many years, is probably the commemoration of the coup, the dictatorship and the resistance."
>
> - Juan (46, Argentine living in Amsterdam)

He also mentioned the commemorative festivity on September 16, *La Noche de los Lapisles* ("the night of the pencils"), which he described as the anniversary of the kidnapping of secondary school students. "And that's a smaller demonstration", Juan said, "but with a very important participation of secondary school, or high school students, that still happens now". The contrast between Juan's experience in the education system and the others' may indicate the results of the different efforts made by Kirchner's recovery-oriented policies.

4.3 Views on the *dirty war* and *two-demons theory*

The *dirty war* narrative and the *two-demons* theory are the ones that sparked the most interesting discussions during the interviews. Firstly, I noticed that the two were used interchangeably by the interviewees, as the distinction between them can be considered quite blurred, and as they were both attributed to the same group of people: more specifically, the right-wing defendants of the military. Maria and Valentina both quickly rejected this narrative when they were asked the question whether they were ever exposed to it. Maria said it was merely based on wrong convictions: the Montoneros clearly did not have the means to handle a war against the army. They were not even terrorists, she said, "they did not do things such as going into cafés and shoot to random people... they didn't do such things." She argued that when she studied the dictatorship in her university, the Montoneros and other leftist revolutionaries were seen as the victims, and mostly as an "expression of the research of a better world, which is

what the sixties and seventies were about, no?". Similarly, Valentina did not see any terrorist trait in the leftist opposition groups, and rejected the *guerra sucia* expression immediately:

> "Videla, who was the biggest devil of all, let's say the mind behind all of this from the beginning, he used to argue that there were many terrorist groups rising, but it wasn't the case".
>
> - Valentina (52, Argentine living in Italy)

Valentina said that they did not live through terrorism, and she compared Argentina's situation to the terrorism that instead was taking place in Spain during those years. She also admitted that being from Mendoza, a town at the same latitude as Buenos Aires but closer to the Chilean border, meant her experience was characterized by less violence and turmoil than that of those living in the capital. Pedro argued that the dirty war narrative was obsolete in his social circles in the 80s and 90s, even if his friends belonged to different political affiliations. He said that mentioning that sort of discourse was not "politically correct" back then, as it would categorize people as supporters of the army; "it was really something that you could not say before". Interestingly, he said that this sort of narrative was significantly more popular later on, during the Macri government between 2015 and 2019. He said that when

> "the right-wing people showed up again, they started to deny that the number of victims was not 30,000, it was 7,000, and that there were also murders in the opposition."
>
> - Pedro (43, Argentine living in Amsterdam)

A similar argument was brought up by Juan, who said that if you do not make sure to victimize the opposition group, you risk going into the *two demons theory*, which has been long discussed among survivor groups. He argued that having to rule it out as a potential explanation is the "cost that comes with the *political victory* of saying okay, we will not accept that this is framed as a war". Juan only came to the Netherlands in 2019 and is the one interviewee that has the most recent and strongest contact with how the events are framed in Argentina. He told me that as of today, the *dirty war* narrative and *two demons theory* are not easily welcomed among the Argentinian public. He said:

> "This is the typical argument that is used by those who want to defend the dictatorship openly. But it's been a losing argument in Argentina in the last 40 years, even the right wing will have to prove... I mean, even the right-wing government of Macri, they really wouldn't try to go into that, because they will make no sense for them, they will lose that argument. So that's really the non-legitimate argument".
>
> – Juan (46, Argentine living in Amsterdam)

He recognized the existence and recent growing presence of what he called "negationists": those who claim that the *desaparecidos* were 7,000 and call out the crimes of the guerrilla groups. However, he said that they struggle to find ground in the public opinion.

Nevertheless, one interviewee, Carlos, exposed extremely different views on such matters. Carlos started the interview by saying that he could give me the "real story" behind the events of the last Argentinian dictatorship, as most people only tell half of the truth. He immediately started the conversation by mentioning the horrors of the end of the 60s and 70s, and the damage that leftist revolutionaries caused to Argentinian society. He listed the revolutionary groups, mentioning the AAA (Alianza Anticomunista Argentina), the ERP, and the Montoneros, and saying that they were nothing else but terrorists. Carlos told me that he moved to Buenos Aires from Cordoba, Argentina, in his 20s to fulfill the mandatory military service. It was during that time, between 1970 and 1972, that he was confronted with very violent episodes where he fought against the revolutionary groups.

> "Oh, yes. They attacked los Cuarteles Militares (the military headquarter). Where the military was... And they attack in the night for example, because they want to take the guns, to steal the guns for the... And then for attacking that people. Yeah, attack to the police too. That is terrorism, pure pure terrorism."
>
> -Carlos (69, Argentine living in Utrecht)

He kept describing the fear and terror of that period:

> "In 70-72 to 76 was a chaos, a chaos, chaos. Buenos Aires, was terrible because you left the home for the war and you didn't know if you're coming back. You didn't know, you didn't know because bombs exploded everywhere and that those bombs were from the earth of the Montoneros. This chaos.. all the people or many people, I was one of them, asked: please, the military has to take the order here in this country otherwise everyone.... *todos vamos a morir* (we are all going to die)."
>
> -Carlos (69, Argentine living in Utrecht)

Carlos saw the military's intervention as necessary and argues that many others did too. When asked to describe what happened in Argentina between 1976 and 1983 in a few sentences, Carlos said there was a war between two groups: the army and the terrorists. Carlos specified that he never agreed with the means that the junta employed against the political opponents during the dictatorship and rejects the torture and mass

killings carried out by the military but was very (even more) explicit in condemning the leftist groups as well. He called them 'monsters' and was visibly upset when talking about how much they are glorified in the country, arguing that "they are monsters, not heroes". He mentioned names of people he knew were part of these groups and told me about the crimes they committed. Carlos also argued that the real number of the *desaparecidos* does not exceed 8,000, and that the main reason why "they created" (as he said) the number 30,000 is because of subsidies that were granted from the Netherlands to former Montoneros, and generally because it was a more appealing number for economic compensations. In short, Carlos' way of telling the story corresponds to what Pedro and Juan would characterize as a far-right wing military supporter. Carlos said that he does not consider himself affiliated with the right, but that there is a lot of ignorance around what truly happened before and during the dictatorship. Carlos left Argentina as soon as the dictatorship was over, and rarely went back.

Because of the limited scope of this research and my small sample, I could not infer whether Carlo's perspective is representative of his age group or whether his opinion is very unpopular among most Argentines, I can only deduct that finding him through Facebook might have contributed to him being an "outlier", as contacting me from his own initiative might indicate a more radical viewpoint. What I could observe is that Carlos' way of narrating the history of the dictatorship is clearly affected by his personal experiences in the army and his little exposure to the narratives that followed afterwards, in contrast to the other participants, as the next few paragraphs will illustrate.

4.4 State Terrorism as central narrative

"This is state terrorism, this was not a war", Juan said firmly when I asked him about his views on the term *guerra sucia*. Moreover, he added that in Argentina the events have been institutionalized in favor of the state terrorism narrative, marginalizing the *two-demons theory* and *dirty war* discourse to people with a specific political affiliation. Maria also said that in her Sociology faculty, the way she studied the dictatorship meant strictly condemning the government for their violent acts and adhering to the frame of state terrorism. Therefore, she said, that is also the perspective she adopted, as she feels that it is in line with her moral and political views; "The Sociology faculty is very left-leaning in Argentina" she observed, and later on she voiced her political affiliation by proudly saying "I'm a leftist person". Furthermore, the state terrorism narrative was preferred by the other interviewees too: Pedro recalled that it was how things were told during his upbringing: "probably because of my family's leftist political orientation", he admitted. Valentina energetically nodded as I said the words "state terrorism"; she responded with "yes, it was such a dark, dark time". Carlos was the only interviewee whose perspective differed drastically from the general consensus, as mentioned in the previous paragraph, and when the term "state terrorism" was mentioned he immediately responded by saying that it was the dictatorship that started in response of terrorism, not the other way around. Therefore, he was the only one among the five interviewees that decided to stick to the *guerra sucia* narrative, attributing the word "terrorist" only to the rebel groups he fought against during his military service in the 80s.

Moreover, as I investigated the usage of the term "genocide" to describe the events of the dictatorship, the response was quite limited from most of the interviewees. Maria explicitly admitted that she was never exposed to it and was quite surprised when I mentioned Robben's observations on the increasing usage of that narrative. She responded: "the difference stands out, doesn't it? Whereas back then they tried to enforce the *Full Stop Law*, now they call it genocide". To Carlos, the number of disappeared was too small to consider the events as a genocide, whilst Pedro attributed the use of this discourse to Kirchner's greater populist plan to gain consensus in the scarred population. The critiques of Kirchner's administrations will be highlighted in the next paragraph. Juan is the interviewee that seemed the most familiar with the term "genocide" to describe the political oppression of 1976-83, probably because he is the one with the most recent experiences in Argentina, while the other participants are more disconnected from their country of origin. Juan affirmed that "[the term genocide] is used, but a bit loosely", and emphasized that, compared to Europe, in Argentina there is less attachment to the history of the Holocaust of the Second World War; therefore, it is somewhat easier to make associations between the events. He said:

> "The Holocaust is very present, but of course, not as present as in Europe. So, the comparison is, if you say that [the *desaparecidos* were victims of a genocide] in a country that experienced the genocide in the 1940s, people will say, well, are you comparing this with that?"
>
> – Juan (46, Argentine living in Amsterdam)

However, he highlighted that he was not aware of any noteworthy debate about whether genocide was to be the preferred term for the Argentinian case. He said that there are more important and frequent narratives: the *Nunca Mas* (never again, which is also the name of the 1984 CONADEP report), "memoria, verdad y justicia" (memory, truth, and justice), or simply condemning it as a violation of human rights. Therefore, using the term "genocide" to describe the case of the *desaparecidos* does not seem to be particularly recognized by my interviewees, who instead (except for Carlos) showed a greater attachment with the narrative of state terror-

ism, recognizing it as the term that best labeled the political oppression of the 20th century. My analysis attributes this general preference to their individual life experiences, which exposed them to this specific narrative more so than the others: those being their families' opinions, their educational environments, or simply the influence of their political orientation. Moreover, Valentina, Pedro, and Maria all left Argentina in the 1990s, when the state-terrorism narrative was already becoming more popular than the *dirty war* or *two-demons* frames. Since Carlos left the country right before the dictatorship ended, he is lacking the exposure to that narrative. My data suggests that the timing of the diaspora members' departure from Argentina plays a role in what narratives regarding the *desaparecidos* the diaspora adopted as their own, as it influences the way they became acquainted with the historical events, and with their societal relevance.

4.5 Transitional justice: CONADEP and trials of the junta

Investigating how the establishment of the CONADEP and the trials of the Junta are still perceived among the Argentinian diaspora was a fundamental part of this research, as I intended to integrate a new understanding of transitional justice strategies, focusing on their long-term impact on individual experiences. I was very surprised to hear that, although all interviewees declared that they knew what the CONADEP was, no one seemed to be fully aware of its centrality in the immediate post-dictatorship phase. My surprise derives from my initial belief that these means of transitional justice were somewhat more publicly recognized, which derives from the importance that Robben's analysis (2018) attributes to them in the process of national recovery. However, the following results show that public opinion – as far as my interviewees reported - adopted different perspectives and critiques.

Juan said he first heard of the CONADEP when studying history at school, and that it is never brought up when the media or politicians talk about the *desaparecidos* in Argentina. When asked about the CONADEP, Carlos replied "first of all, I want to have another commission" that could study and shed light on the crimes committed by the rebel groups, because they have been wrongly glorified and little is known about their actions. Pedro explicitly admitted that he had no opinions about the truth commission, and both Maria and Valentina acknowledged its existence, but quickly moved on to talking about the trials of the junta, which was instead a topic that prompted more observations from all interviewees.

When talking about the trials of the junta, the conversations were characterized by two main arguments: firstly, there was some general criticism towards the amnesties and the fact that they let many officers get away with their crimes. Juan's words summarize this point very well:

> "in any case, it was long time ago. And the big problem of the Judicio de las Juntas is that two years later they stopped all the trials because of these two laws [here he is referring to *Ley de Punto Final* and *Ley de Obediencia Debida*]. And then six years later, they pardoned the junta's leaders. So it kind of lost some of the appeal. Because... first, yeah, it went well, and then, after a couple of years, there were many setbacks".
>
> – Juan (46, Argentine living in Amsterdam)

When questioned about the trials, both Maria and Valentina immediately mentioned and criticized the "indulto", which refers to Menem's pardon of the military officers and of the generals Viola and Videla. This shifting focus shows that there was more interest on how poorly the trials were handled in the late 1980s, rather than the fact that they effectively took place later on. Besides, the second recurring argument, which was brought up by all five interviewees, attributed the trials at the beginning of the 2000s to Kirchner's political plan to reach national consensus. Everyone saw Kirchner's decision of re-starting the trials as unnecessary: as many years had passed from the events, most of the officers were extremely old and many had died already. Pedro said with a contemptuous face: "they had no more power... no more political, military or any sort of support, so it was easy to catch them", and later added "it was a typical populist act, they did things that they thought the people are gonna like, and they did get on with those things". With "they" Pedro refers to the Kirchners: therefore, president Néstor who ruled from 2003 to 2007, and Cristina Fernández de Kirchner, whose mandate lasted for the two following administrations (2007-2015) and who has been the vice-president of the country since 2019. Taking the same critical perspective, Maria mentioned that the trials in the 2000s were strategically planned to appease the middle class, which she described as particularly influential in Argentina. Carlos, who vehemently stated that he "one hundred percent rejects the Kirchner government", said that the trials happened only because the Kirchners "aprovecharon" (took advantage of) the situation to become extremely rich. Valentina described the trials of the 2000s and the way they affected Argentinian society by using the metaphor of someone who is hungry and does not know how to fish in order to feed themselves. "The Kirchner gave them (the Argentines) a piece of fish to eat but didn't teach them how to fish by themselves", she commented, and also said that it is typical for the Argentinian mentality to "always and only get the fish", which she explained as trying to solve big problems with quick and short-termed solutions. In her view, trialing the old junta members was a way to ap-

peal to the public without making more socially impactful changes. Maria made a similar point when she eloquently argued: "social processes are not accomplished through a law or a decree, social processes are stronger and longer, they go beyond that".

I believe that the tendency to confine Kirchner's actions to "populist strategies" is related to the interviewees' disappointments with the recent Argentinian governments. All of them regard the last few administrations as harmful for the country, and as the causes for its economic and social problems. It is quite logical that as leaders lose credibility with failed interventions, their actions lose recognition. Since the Kirchners are seen as not able to improve Argentina's deep-rooted issues ("they can't teach how to fish"), they are also not regarded as capable of positive changes in regard to its troubled history, from which they can only profit ("aprovechar") and use in their favor.

4.6 The *desaparecidos* as a topic of conversation

A fundamental part of my study aimed to discover how the topic of the *desaparecidos* is treated by its diaspora, both within Argentinian communities abroad, and between Argentines and Europeans. I initially hypothesized that talking about it to compatriots or to people who are not familiar with it would be a form of attachment to their country's history and an indicator of entanglement between traumatic history and national identity. Therefore, I inquired whether the events of the 1976-83 were something that brought Argentines together when abroad, for example in the form of collective victimhood. Nevertheless, none of the interviewees claimed to be an active member in any diasporic community, and they reported barely having Argentinian friends.[2] Pedro even said that he "avoids Argentines abroad" as he had "bad experiences". This comment is quite relevant, as it shows how Pedro decided to distance himself from a community he formerly belonged to. He made generalizations by saying that all Argentines are characterized by a specific mentality, which he feels superior to, and adopts the perspective of an "outsider" by being wary of his association with compatriots abroad. Similarly, Valentina did not manage to connect with Argentines since she moved to Italy; she said they are too "nostalgic", idolizing a country that she does not recognize the way they do, saying that "they didn't live through the economic crisis and the troubles we [she and her husband] went through". Like Pedro, Valentina distanced herself from the other diaspora members, seeing herself as more aware of what "back home" truly means, with the poverty, suffering, and instability present in Argentinian society. Maria said that she used to see a group of compatriots in her first years in Italy, but then mostly stopped as she got busier with her children and family. "No", she said, "I would never talk about the *desaparecidos* with them... it's something that you can bring up only in very specific environments". When I asked what ties her and the other Argentines together, she replied: "the food, the songs... things like this". For Maria, the artistic heritage of her culture is the only thing binding her to her compatriots; otherwise, she distances herself from them, prioritizing her family's needs and, as she said, a more "Italian" lifestyle. The political oppression of the last dictatorship is only regarded as a historical chapter of her country of origin, which does not impact her attachment to it, nor her national identity. Carlos said that he only has one Argentinian friend in the Netherlands, where he has lived for over 30 years; with her, he said he would never bring up the topic of the *desaparecidos*. He said "I am (politically) too 'right' for those people", and he is afraid of being seen as a radical supporter of the military. On the other hand, Juan said that with the few Argentines he has met during his three-years stay in the Netherlands, the events of the dictatorship are not a divisive topic: "everyone seems to be on the good side", he said, it's not like "some people say well, you know what, I hope the military comes back and kills everybody in Argentina" he joked. However, Juan still stressed that the topic itself never comes up. "Time is passing by" he told me, acknowledging the importance of time in changing the way history is perceived. When talking about back home, there's more important and urgent things to talk about, such as the Kirchners' governments, which Juan described as the most divisive one, since people tend to have more contrasting opinions about it. Therefore, all interviewees reported different, and almost opposite, reasons as why they do not approach the topic of the *desaparecidos* with other Argentines abroad: either because it's considered tragic and polarizing, or because it is considered unnecessary and tied to a distant past.

Next, I inquired whether the last military dictatorship was ever a topic of conversation with Europeans when my interviewees spoke about their home country. The five interviewees reported that mostly it is simply not something they refer to when they describe Argentina. Pedro laughed, saying "they (his European friends) usually want to talk about Maradona" and added that at times he happens to answer questions about more recent politics. Maria said that when she arrived in Italy in 2002, a destructive economic crisis hit Argentina; the crisis became the last mental picture she has of her home country, and what she refers to when talking about it. Similarly, Valentina mentioned that when she describes Argentina to her European friends, she can only think about the few opportunities that there are, the poor healthcare that made many of her friends suffer or die, or the social and economic precarity that the growing poverty has been causing on its greatly unequal society. Juan said that in the Netherlands, people

[2] Note that the Facebook group in which I found some of the participants was not a diasporic community. Instead, it was mainly used for questions about Dutch bureaucracy and there was only a loose social bond between the members.

are more or less aware of that part of history because of the protests that succeeded queen Maxima's coronation in 2013 (which sparked some discussions about the *desaparecidos* and her father's involvement in the junta's government). However, the political oppression is still not something that will be brought up in most of his conversations about Argentina. The fact that members of the diaspora rarely engage in this part of their country's history as a topic of conversation – both with other Argentines and with Europeans – shows that there is little individual effort to propagate this collective memory, or at least their personal account on it. Furthermore, it seems that more recent events and socio-economic problems in Argentina gain priority over the events of the late 1970s and 80s, which are now perceived as "far away" from the current Argentinian reality. Now that the injustice surrounding the *desaparecidos* has been acknowledged and recognized, there are more pressing issues that require space and attention. While conducting the interviews, I observed that the events we were discussing were considered part of some old history by most of the participants; during the interview, Juan mentioned multiple times that a lot of time separates those events from contemporary matters. Interestingly, all of them (but Pedro) at some point during their interviews spontaneously referred to the *desaparecidos* as an "open wound" for Argentinian society. The "open wound" metaphor has been used in the literature to refer to *desaparecidos*' history (Crenzel, 2020), and might have become a recurring figure of speech in the Argentinian public discourse. The open wound metaphor indicates the causes of ongoing suffering, something that has not found healing and therefore still damages the societal fabric of Argentina. When I asked Valentina whether the wound would turn into a scar, she said with a serious face "it is not a nice thing if the scar is created, the scar shouldn't be there, so that the injustice we lived won't be forgotten". By using this metaphor, the interviewees showed awareness about the essential role that the events of the *desaparecidos* have had in the development of Argentinian history; however, this seemingly objective claim goes along with personal detachment from the events themselves.

Perhaps, the interviewees' detachment from the injustice of the last dictatorship comes from the privilege of not having been personally involved in the traumatic events of the *desaparecidos*. Close involvement would have resulted in different attitudes toward its societal consequences and public response. Moreover, according to my observations, the physical and geographical distance between the diaspora and their country of origin was complemented by the socio-economic distance that living in Europe implies. Living in the "rich" West has allowed them to compare the economic states of their original and host country, focusing on Argentina's material problems and obscuring other societal issues that are just as present or relevant.

4.7 Discussion

The research question of this study attempted to unpack the way Argentines living in Europe narrate the events of the last military dictatorship, and the role that these play in their national identity. The departure point of this study was the theoretical framework centered on collective memory and socio-cultural trauma, together with the six narrative frames identified by Robben (2018): denial, secrecy, dirty war, two demons theory, state terrorism, and genocide. These frames were later used as instrumental points of analysis for the qualitative study conducted with the Argentinian diaspora in Europe. The results of this study show that state terrorism is the preferred interpretation of the 1976-83 events, as it was the main discourse supported by four out of five interviewees. Only one interviewee showed a contrasting inclination, preferring instead the *dirty war* narrative. I hypothesized that such a difference might be related to the different life experiences (the only exception, Carlos, has worked in the army prior to the dictatorship) and to the different time periods in which the study participants left Argentina. Other significant differences were observed between the four older interviewees, who have lived in Europe for at least one decade each, and Juan, the youngest, who instead moved to the Netherlands from Argentina only three years ago. Juan seemed more informed, more engaged, and aware of the societal implications of the last dictatorship in Argentina. Furthermore, my results indicate that the history of the *desaparecidos* is a topic that the participants of this research rarely engage in – both with other Argentinians and with Europeans – therefore producing little to no propagation of their account on this collective memory. They all showed to be knowledgeable and to have strong opinions about the topic, but they revealed little attachment to the issue at a personal level, as it seemed to have little impact on their national identity. The interviewees recognized other elements as characteristic of what they would regard as Argentinianness: they highlighted some characteristics of the "Argentinian mentality", showed fondness for their folkloric culture, and largely criticized the country's recent governments. In combination with emotional distance from the *desaparecidos* history, the interviewees showed more concern regarding the recent socio-economic issues present in Argentina, such as economic instability, an increasingly divided society, and insufficient job opportunities.

These results differ from my initial convictions which, influenced by little exposure to Argentinian society and merely shaped by what I studied for this research, saw the topic of the *desaparecidos* as a central constituent of Argentinian national identity. The works by Robben (2012, 2018), Edelman et al. (2003), Stockwell (2011), and Crenzel (2020) outlined in the theoretical framework described the events around the *desaparecidos* and the socio-cultural trauma deriving from it as a permanent mark on Argentinian society. I attribute the dis-

similarity between my research's results and the aforementioned literature to the essential difference in our subject of study: here, the Argentinian diaspora has the fundamental feature of being physically distant from its country and society of origin. The observed distance from the *desaparecidos* by the interviewees might be related to the fact that they have been *deterritorialized* (from Argentina) and *reterritorialized* (in their corresponding European countries). Deterritorialization and reterritorialization are terms coined by Deleuze and Guattari in their work *Anti-Oedipus* (1977), which defines the two terms as "reorganization - a debordering and rebordering - of territorial power, autonomy, and identity at multiple scales of government, from the global governance system to the local state" (Woodward & Jones, 2017, p. 35). They introduced the concept to explore fundamental societal issues of the contemporary world, including globalization, the prominence of capitalism, and migration flows. They also discuss the importance of deterritorialization in the formation of fluid identities, which they see as the result of bodies that are transported in different geographical locations with different socio-historial realities, giving rise to a "system of interiority" which re-organizes the way space and identity are perceived (Deleuze & Guattari, 1977). In his essay *Disjuncture and Difference in the Global Cultural Economy*, Arjun Appadurai eloquently builds on the notion of deterritorialization by arguing:

> "Deterritorialization, in general, is one of the central forces of the modern world because it brings laboring populations into the lower-class sectors and spaces of relatively wealthy societies, while sometimes creating exaggerated and intensified senses of criticism or attachment to politics in the home state." (Appadurai, 1990, p. 35)

Such phenomenon corresponds to the results of this study, where the participants showed vivid criticism of Argentina's political situation, reproach for what they regarded as "Argentines' mentality", and even explicit disapproval of the Argentinian diaspora they know.

One of the most important weaknesses of this study is the fact that I could not expand on noteworthy topics that emerged during the interviews. For instance, Valentina presented an argument that diverted from the six reference frames I used to interpret the *desaparecidos* history: she said that her parents long argued that Argentina and Chile's dictatorships were strategically orchestrated by the United States, which saw the two South American countries as a threat to their economic superiority. I decided to not include this narrative in the analysis, as it was an isolated argument that would have required further examination, going beyond the scope of this short research study. Another thought-provoking argument that was excluded in the analysis was brought up by Juan, who mentioned that among the societal consequences of the *desaparecidos*' history, the most present in Argentinian society is an enhanced sensitivity to police brutality. More generally, he argued, people seem to be particularly perceptive of state violence, and institutional distrust is widespread in the public. Juan's claims correspond to Robben's (2018) account on the effects of socio-cultural trauma on trust/distrust in relationships and align with Edelman et al. (2003)'s observations regarding skepticism and disillusionment. Despite the very intriguing nature of this argument, it is based on the domestic Argentinian social framework, deviating from the focus on Argentinian's diaspora, which this research focuses on. Other weaknesses of this study concern the small sample size of only five participants. If I had more time and opportunities to interview more members of the Argentinian diaspora, different trends might have arisen, and different life experiences might have resulted in novel perspectives over the topics we discussed. Moreover, as most of the interviews were conducted online, the one-to-one rapport has suffered from the lack of physical proximity, which I perceived as a communication facilitator with Valentina, the only interviewee I met in person.

5 Conclusion

This study investigated how members of the Argentinian diaspora in Europe narrate their national identity in relation to the events of the *desaparecidos*, which occurred in Argentina during the last military dictatorship (1976-83). The research question was answered by breaking down the complex matter into three main steps. Firstly, the study provided an overview of the historical context of the topic of research, and the theoretical framework that represented the departure point for the investigation. Secondly, the narratives that domestically emerged in Argentina during and after the dictatorship were laid out, and lastly, the qualitative study analysis followed. The latter included five participants, all members of the Argentinian diaspora living in Europe (or who have lived in Europe), whose ages ranged from 43 to 69 years old.

The study's results can be divided into two sections: narration and identity. The analysis of the narratives employed by the participants led to the conclusion that generally, the narrative of state terrorism is the one preferred among most people I interviewed, as opposed to the other discourses of silence and denial, *guerra sucia*, *two demons theory*, or genocide. Four out of five interviewees tended to victimize the rebel groups and attribute the "perpetrator" role to the military in charge during the dictatorship. Only Carlos took a different stance: although he recognized the junta's wrongdoings, he stuck to the *dirty war* narrative. This choice is probably related to his former military affiliation and to the fact that he left Argentina right as the dictatorship ended.

Nevertheless, the most important observation of this

study is that individuals' views and opinions are majorly shaped by life experiences and personal exposure to different interpretations of the events, which are in turn influenced by the time in which the diaspora members left the country of origin. These differences can be summarized by the following sentence: there is no uniform perspective among Argentinian diaspora members, as people who have left Argentinian territory differ in the way they narrate the events, and how they are attached to collective memories and national history is extremely variable and dependent on their individual experiences. The main means of transitional justice delineated in the first half of the study, namely the CONADEP and the trials of the junta, were not recognized by the interviewees as leading measures for national healing. The trials of the junta were instead a source of criticism toward the amnesties of the late 1980s and '90s and toward the Kirchner administrations.

Regarding the participants' national identity, the study shows that the history of the *desaparecidos* is of limited relevance for the diaspora's identity formation. The participants showed little interest in propagating their memories to other diaspora members or European acquaintances and demonstrated more attachment and concern for the current government and socio-economic issues in the country. Future research could further investigate my research question through studying a larger sample size, therefore allowing for comparison between the different European "host countries" and between diaspora members of different age groups or from different socio-economic status. Moreover, Deleuze and Guattari's (1977) concept of deterritorialization should be taken as the central theoretical point of analysis to explore the influence of physical distance on collective memories' preservation and nurturing, together with its impact on the emotional attachment to the diaspora's original country and culture. Furthermore, qualitative fieldwork in Argentina could explore the relationship between the "open wound" left by the *desaparecidos* and popular institutional distrust in the country, a subject briefly pointed out by the interviewee Juan. Lastly, the *desaparecidos'* history and its social consequences beyond the Argentinian border, impacted by the presence of the diaspora abroad, is a fascinating yet understudied field of research in need of further exploration, which has the potential to provide social sciences literature with major insights about distance, collective memory, and perception of social injustice.

Acknowledgements

I would like to express my deepest gratitude to my thesis supervisor Dr. Younes Saramifar, who has been a source of great support and a familiar guide. He helped me out throughout the whole semester, he showed flexibility when I needed some, taught me the workings of qualitative research and believed in my capacities whenever I felt overwhelmed by this project. I am grateful for my reader, who still managed to dedicate time and commitment to my research, despite being geographically far and extremely busy with her own research. I also thank her for being a caring teacher, and for her support even outside of the classroom. Moreover, I thank the participants of this study, who have been extremely helpful and available, and approached our interviews with remarkable trust and openness. I would like to thank my family, who has always sustained and encouraged my academic efforts and achievements, and whose care and affection I cherish and treasure. I would also like to thank the team of peer-reviewers whose work made this thesis improve substantially. Miriam, Chynna, Sophia, Shree and Jannik, thank you for your time and your interest in my words, I'm very happy with what we produced together! Lastly, I am profoundly grateful for my amazing friends, with whom I shared both the most beautiful and the hardest moments of the past three years, and whom I now regard as my second family. We embarked on the capstone process together, and – through delicious pastas and deep-focus playlists - overcame it together. Thank you.

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Humanities

Commemorative Processes and Slavery Memorials in the Netherlands

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Abstract

Although extensive research exists on public displays of Dutch colonial history and the Netherlands' involvement in the global slave trade, there is currently a gap when it comes to investigating slavery memorials as a larger, Dutch racial redress trend. This research uses James E. Young's criteria for successful memorial processes to argue that the slavery memorials in the Netherlands constitute a successful racial redress trend, as they join civil society agents, impacted communities and municipalities/the Dutch state in conversations about how to challenge a hegemonic narrative of Dutch slavery history. The discussions of existing and planned slavery memorials in this thesis base themselves on elaborate research of primary source materials from the online archives of municipalities, newspapers and civil society organizations enriched by the conduction of qualitative dialogic interviews with stakeholders involved in ongoing memorial processes. This research shows that the emplacement of slavery memorials into visual spaces in the Netherlands counters hegemonic narratives of Dutch slavery. Furthermore, it concludes that the memorial processes result in the creation of specific sites for Keti Koti commemoration and celebration, which makes the functionality of the physical memorials significant for fostering discussions of Dutch slavery history even after the monument inaugurations.

Keywords and phrases: *Racial redress, hegemony, slavery, memorial, memorial process, James E. Young, Netherlands*

Suggested citation:
Hansen T.B. (2022). Commemorative Processes and Slavery Memorials in the Netherlands. *AUC Undergraduate Journal of Liberal Arts and Sciences*, 19(1) pp. 106-125

imprint.

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1 Introduction

In 1993, which marked the 130^{th} anniversary of the official abolition of slavery in the Netherlands and the 120^{th} anniversary of the actual end of the Dutch slave trade (Cain 228), the *Nationaal 30 juni/1 juli Comité* was established by Amsterdam citizens of African and Surinamese descent (Kardux 171). This marked the beginning of a wave of racial redress activities in the Netherlands, and around 60 racial redress initiatives have followed since then in an attempt to solidify Dutch slavery past as an integrated aspect of Dutch national history (NIOD data). These activities often constitute examples of combined efforts between civil society organizations and politicians, municipalities and/or the Dutch state. Especially after the 2002 erection of *Het Nationaal Monument Slavernijverleden* in Amsterdam, anniversaries and decennials of the abolition of slavery in the Netherlands, known as Keti Koti, are celebrated annually on 1 July and have become more prominent, first in Amsterdam and later in other major Dutch cities such as The Hague, Utrecht and Rotterdam (NIOD data). Following the inauguration of the monument in Amsterdam, Middelburg and Rotterdam erected memorials for commemoration of the Dutch slave trade in 2005 and 2013 respectively (Koops, BKOR "Slavernijmonument"). In connection to the upcoming 150^{th} anniversary of Keti Koti in 2023, The Hague, Groningen, and Utrecht have announced that they will all erect local slavery memorials, and Tilburg has already planned to erect one on Keti Koti 2022 (Rubio, Scheffer, NOS, Hest).

This research takes its starting point in historian James E. Young's analysis of the inherent problems connected to situations in which nations attempt to display non-favorable aspects of their history through memorials (Young, "Counter-Monuments" 270) in order to examine Dutch slavery memorials. Young focuses immensely on the significance of the *process* when it comes to erecting memorials and emphasizes that

> "dissent and debate [are] also part of the process and (. . .) any memorial worth realizing [has] to be capacious enough to contain and manage the needs of both its advocates and its detractors, both its local and its national constituency" (Young, "Stages of Memory" 189).

Young thus stresses that debate potentially preserves the memory, which a memorial attempts to eternalize, more successfully than the actual physical memorial. However, the connection between Keti Koti celebrations and the erection of Dutch slavery memorials highlight the importance of the physical monument, which tends to be neglected by Young. This research thus aims to approach the processes of erecting Dutch slavery memorials through Young's methodology, but at the same time seeks to avoid disregarding the spatial importance of the memorials.

The case studies of the three already erected Dutch slavery monuments, that is: *Het Nationaal Monument Slavernijverleden* in Amsterdam, *Het Zeeuws Slavernijmonument* in Middelburg and *Slavernijmonument Rotterdam*, build on multifaceted source material selected from online municipality archives, online newspaper archives and websites of civil society organizations involved in the memorial processes. Since this analysis aims to investigate the roles of various stakeholders, especially the Dutch state, municipalities, and civil society organizations, in the planning and inauguration of slavery memorials, the selected and applied sources rely on information from four angles. Websites and online archives of the following information categories provide these different perspectives: civil society organizations (petitions and websites), local news (focused on the municipality or even districts within municipalities) and national news (national Dutch newspapers), municipalities (websites, press releases and covenants) and academic (scholarly sources and research institutes). In addition to this, qualitative research in the form of interviews with key stakeholders of the processes in Tilburg, Utrecht, and the Hague constitutes primary source material for the discussion of not yet erected slavery memorials.

This research argues that the slavery memorials in the Netherlands constitute a successful racial redress trend, as they bring together civil society agents, impacted communities, and municipalities and/or the Dutch state in conversations about how to challenge a hegemonic narrative of Dutch slavery history. Furthermore, this analysis explores how the emplacement of slavery memorials into visual spaces counters this hegemonic narrative and establishes that the memorial processes result in the creation of specific sites for Keti Koti commemoration and celebration, which makes the functionality of the physical memorials significant. To commence, existing research on public displays of slavery history in the Netherlands will be explored. Then follows an analysis of the slavery memorials in Amsterdam, Middelburg, and Rotterdam as well as a contextualization of the monuments within the Dutch racial redress debate and the tendencies detected among redress activities. This paves the way for a discussion of how the already erected monuments function as counter-hegemonic visual source material when emplaced within hegemonic visual narratives in Dutch city spaces. Finally, this research examines the current wave of local monument erection processes through interviews with key stakeholders in order to discuss potential continuity between already erected and planned monuments for slavery commemoration in the Netherlands. It concludes with a reflection on the success of slavery memorials as a redress trend.

2 Visualizing Dutch Slavery History

The various processes for erecting national and local slavery memorials in the Netherlands are here described as an example of a Dutch trend for 'racial redress' activities. 'Redress' is interpreted in alignment with the International Commission of Jurists' (ICJ) definition as explained in the guide *The Right to a Remedy and Reparation for Gross Human Rights Violations* from 2018 and the definition presented by the United Nations' (UN) Committee against Torture (CAT) in *Convention Against Torture and Other Cruel, Inhuman or Degrading Treatment or Punishment* from 2012. According to the UN CAT, 'redress' contains all forms of, "effective remedy and reparation [. . .] and refers to the full scope of measures required to redress violations" (UN CAT Article 14, point 2). In that sense, the term 'redress' acknowledges that a violation has happened without dictating the manner in which restoration should take place. Initiatives referred to as 'racial redress activities' can thus combine what the committee calls "reparative concepts" without restricting restoration to one specific concept, for example monetary compensation (ibid.). The ICJ's definition adds that the terms 'remedy' and 'reparation' often limit action to a state level and a "notion of State responsibility" (ICJ 30). 'Redress', on the other hand, manages to encompass multiple and interlinked types of reparations while simultaneously not excluding non-state actors from the reparation process.

Another terminological decision relates closely to the case studies, as this research adopts Young's idea of a 'memorial' as something dynamic and living that exists within a city space, whereas he defines a 'monument' as "authoritarian", "fixed" and "static" (Young 4, 6). In order to emphasize how the Dutch projects for visualizing slavery in public spaces differ from more traditional monuments such as war commemoration monuments and statues depicting specific people, this research consequently refers to them as 'slavery memorials'.

2.1 Hegemony and the Hegemonic Visual Narrative

This research frequently employs the term 'hegemony', which makes it important to stress which definition, or perhaps rather, understanding, of the term applies here. Arguing for a hegemonic Dutch narrative of slavery in the Netherlands may seem potentially problematic due to the difficulty of determining exactly *what* this supposed narrative entails. According to the prominent scholars of the Dutch slavery past and its commemoration, Paul Bijl and Artwell Cain, a narrative which downplays the violence of the Dutch slave trade constitutes a possible working definition of the term 'hegemonic' (Bijl 458, Cain 229-230). Bijl sees the creation of a hegemonic Dutch narrative of slavery not as a master-narrative attempting to delete minority experiences from Dutch history, but rather as a dominant narrative which does not view victims of Dutch colonialism as "memorable within a national context" (Bijl 458). Similarly, Cain argues that this hegemonic narrative cannot contain the Dutch participation in slavery and that two main arguments characterize this hegemonic narrative as well as the academics, journalists, and politicians who defend it: firstly, diminishing the Dutch slave trade's scope and distancing the economic success of the Netherlands during the $16^{th} - 17^{th}$ centuries from profit derived from the slave trade and, secondly, refusing to acknowledge the negative experiences of contemporary descendants of enslaved people and members of minorities from former Dutch colonies which connect to slavery history and the commemoration thereof (Cain 229-230).

Young and the cultural historians Andreas Huyssen and Jennifer Tosch all provide a more multifaceted interpretation of 'hegemony' than Bijl and Cain, as they view visual hegemony as a question of what one focuses on rather than as a master-narrative, which needs a competing narrative in order to be altered. Huyssen opposes the very idea that a certain unchanging hegemony can persist or has historically existed without changing over time and according to "spatial practices" – the manner in which architecture, culture, and also monuments change spaces (Huyssen, *Other Cities* 3). An example of this could be the placement of a slavery memorial within a space which connects to the Dutch colonial era, e.g. a harbor area or a warehouse, since these spaces were used in the triangular trade. Similarly, Tosch argues that one does not have to "change history" or provide an "alternative history" in order to incorporate minority experiences into a hegemonic visual narrative, since a city like Amsterdam carries traces of minorities everywhere (Tosch 11). Young goes so far as to argue for the impossibility of determining national hegemonic narratives and claims that memorial artworks in public spaces create fixed points of communion for communities that are increasingly characterized by a lack of unity:

> "[In] sharing common spaces in which we collect our disparate and competing memories, we find common (perhaps even a national) understanding of widely disparate experiences and our very reasons for recalling them" (Young "The Stages of Memory" 15).

According to Young, contemporary visual memory work in cities thus undermines attempts to maintain visual hegemonies if the memory work manages to strive for inclusivity and ambiguity rather than a singular, hegemonic narrative of its own (Young "The Stages of Memory 14-15).

Huyssen, Tosch and Young do not argue against the existence of hegemonic narratives, but they all object to the legitimacy of this hegemony, as they show that counter-hegemonic narratives already exist within hege-

monic narratives, waiting to be recognized and made visible. Huyssen writes that,

> "The same space cannot possibly have two different contents. But an urban imaginary in its temporal reach may well put different things in one place: memories of what there was before, imagined alternatives to what there is" (Huyssen, *Palimpsests* 7).

Huyssen thus employs the metaphor of a palimpsest to describe city spaces – just like a surface inscribed with various texts on top of each other, the hegemony of the city (interpretable as the most visible piece of writing on the palimpsest) may obscure the layers below the most visible top layer, but this does not remove the other layers from the narrative. Additionally, providing the counter-hegemonic visual narrative does not, according to Huyssen, imply that a space can be both hegemonic and counter-hegemonic at the same time, but rather that the narrative derived from visual sources of a space changes according to which aspects of the space one focuses on.

So, when this research refers to something as a 'hegemonic' narrative of Dutch slavery, it aims to associate 'hegemony' with the notion that the violence of slave history remains in the past and with the refusal to recognize that unflattering aspects of the Dutch history of slavery have been made less visible in the hegemonic visual narrative of the $17^{th} - 19^{th}$ century Netherlands. The term 'counter-hegemony' will thus refer to initiatives striving to broaden the debate about Dutch slavery history and its commemoration, especially in relation to how visual sources in public spaces affect public discourse. This research interprets 'counter-hegemonic' as increasing the visibility or adding visual representations of narratives, which already exist in the visual source material of a city but remain overlooked in the hegemonic narrative of the city.

2.2 The Academic Debate on the Visibility of Dutch Slavery History

In the article "The Counter-Monument: Memory Against Itself in Germany Today" Young describes the difficulties connected to inaugurating national monuments which commemorate atrocities committed by a nation in its past (Young, "Counter-Monuments" 268). Young writes that:

> "After all, while the victors of history have long erected monuments to remember their triumphs, and victims have long erected monuments to recall their martyrdom, only rarely does a nation call on itself to remember the victims of crimes it has perpetrated. Where are the national monuments (...) to the millions of Africans enslaved and murdered?" (Young, "Counter-Monuments" 270).

Although Young's case studies focus on counter-monuments in Germany and despite there being a lack of counter-monuments in Dutch commemoration activities, Young's analysis of the challenges of national monuments seems to resonate with a central question in the academic debate concerning the commemoration of Dutch slavery, namely: How can slavery memorials contribute to altering hegemonic narratives of Dutch national history, and what are the challenges of getting states or municipalities to support public acknowledgements of slavery commemoration?

First of all, an ongoing academic dispute concerns itself with the question of whether one can refer to the Dutch slavery past as 'forgotten' or 'hidden'. Markus Balkenhol argues for a more complex interpretation of how the Netherlands has historically commemorated slavery as he aims to illustrate how even violent aspects of the Dutch slavery past are sometimes voluntarily displayed (Balkenhol, "Politics of Compassion" 278). While not following Balkenhol's claim that Dutch colonial violence has, historically, been deliberately made visible, Bijl does refute the notion that the Dutch hegemonic narrative of slavery is the product of a deliberate attempt at removing problematic aspects of Dutch national history (Bijl 441).

Cultural historian Johanna Kardux emphasizes the connection between citizenship and commemoration of slavery. She argues that what the French memory scholar Pierre Nora deemed an "exaggerated focus on memory" and sites of commemoration actually constitutes a way in which the hegemonic narrative, which does not emphasize the crimes committed by the Netherlands during slavery, becomes renegotiable (Kardux 166). So, by adding slavery memorials to public spaces, the persistence of the hegemonic narrative becomes an impossibility, because minority memories and alternative interpretations of Dutch history openly challenge this narrative. In another article, Balkenhol refers to this idea of piercing the hegemonic narrative with sites of commemoration as "emplacing slavery" (Balkenhol, "Emplacing Slavery" 135). Emplacing commonly refers to structures on or in which something heavy is placed and thus the introduction of slavery commemoration (for example, the erection of slavery monuments) becomes a means through which minorities within Dutch society can strategically implement more nuanced interpretations into the hegemonic narrative of colonialism and slavery (Balkenhol, "Emplacing Slavery" 141).

Jeroen Dewulf, a Belgian scholar specialized in Dutch language, slavery, and culture studies, views the incorporation of minorities into a hegemonic narrative through monuments as an approach very central to Amsterdam (Dewulf 245). He views this approach as an attempt to work through public trauma and to "preserve public harmony" and he goes even further to propose that diversifying sites of public commemoration could change the idea that Dutch identity separates itself from

the identities of minorities and could replace it with an identity focused on "interaction-in-diversity" (Dewulf 251).

Several other scholars focus on how the constant presence of monuments favoring certain colonialist narratives affect a city space and, more importantly, public discourse. The Dutch professor of modern European history, Michael Wintle's analysis of the "visual history" of Amsterdam provides an excellent example of this, as it focuses on representations of other countries and cultures in Amsterdam through monuments and decorations in the city space. He argues that this representation presents Amsterdam as superior to these foreign countries and cultures (Wintle 81). This, Wintle states, connects to Michael Billig's concept of 'banal nationalism' – the automatic reinforcement of a hegemonic national narrative through the constant 'flagging' of nationalism in the background (Wintle 80, Billig 39). Consequently, according to Wintle and Billig, this presence of small and often not actively recognized remnants of colonialism and colonial narratives of Dutch superiority affect public discourse, since they create a public space which tells a 'visual history' of a hegemonic narrative of Dutch colonialism. In that sense, Dewulf seems to build from where Wintle's analysis ends – namely by arguing that adaptations to public space in Amsterdam, which aim to foster inclusivity and diversity, contribute to challenging the hegemonic visual history created through the presence of banal visual nationalism.

Tosch, who founded the Black Heritage Tours in Amsterdam in 2013, disagrees with Wintle's conclusion that background displays of colonialism re-iterate a hegemonic narrative by bringing attention to exactly these displays of banal nationalism in order to illustrate how people of colour have an inherent claim to be seen as fully fledged citizens of the Netherlands, since the visual history of Amsterdam documents their historical presence in the country (Wintle 81, Tosch vi). Toschalso counters the arguments proposed by Bijl and Cain, since she uses the visual history of a hegemonic nationalist narrative as proof of how Dutch identity cannot be defined as fundamentally one thing, since it intersects with all the other cultures that were present during Dutch colonialism.

Evidently, the vivid academic debate about displays of Dutch colonialism and proposed means for countering a hegemonic narrative of Dutch national history mainly focuses on Amsterdam and not on the broader tendencies of slavery memorials which have been erected in the Netherlands since 2002. With a new wave of slavery monument erections approaching in 2022 and 2023, this research seeks to fill a gap in the academic debate by arguing that the existing and planned Dutch slavery memorials function as local expressions of a larger dynamic of civil society and municipality-led interferences with a hegemonic Dutch narrative of slavery. This is especially prominent when the processes leading to their inaugurations accommodate both civil society agents such as descendant organizations and political actors such as municipalities. This research furthermore aims to locate the slavery monuments within the ongoing Dutch redress debate since 1993 in order to discuss whether the monuments and changes fit into this debate.

3 Slavery Memorials in Amsterdam, Middelburg and Rotterdam

Although Young in his 1992 essay "The Counter-Monument: Memory Against Itself in Germany Today" had trouble imagining that nations would finance erections of slavery memorials, his approach to analyze visual memorial work, as presented in his 2018 essay collection *The Stages of Memory*, provides the fundamental method for the following analysis of the processes leading to the unveilings of the three already erected Dutch slavery memorials. *The Stages of Memory* argues that successful visual memorial work functions as processes full of unresolved conflicts and debates, which continue after the erection of a memorial, rather than culminate with the unveiling of a memorial (Young "The Stages of Memory" 7). In order to solidify this argument, Young writes, "Better a thousand years of Holocaust memorial competitions in Germany than a final solution to your Holocaust memorial question," referring to the 1995 competition to find a design for the memorial for the murdered Jews of Europe (Young "The Stages of Memory" 7). Thereby, Young argues that a design competition for a significant memorial should not strive for a perfect solution that everyone agrees on. Instead, it should focus on understanding the importance of a continuing debate surrounding the memory represented or visualized by the memorial even after its physical completion.

In order to follow Young's argument, this analysis of the three existing slavery memorials will not go into details about the artists behind the memorials or their visual designs. Instead, this analysis focuses on the memorial processes themselves in order to evaluate these according to Young's success criteria and eventually place the memorials within a broader context of Dutch racial redress.

Young determines the success of a memorial process based on the following main criteria: the process must accommodate disagreement between stakeholders, the final memorial should reflect the existence of disagreements rather than attempt to depict a unified or hegemonic narrative of what it represents, and that the memorial, "might also be regarded as a never-to-be-completed process, animated (not disabled) by the forces of history bringing it into being" (Young "The Stages of Memory" 16). Young divides the memorial process into several stages, but since the discussion phase

is the prime focus of this research, only what Young terms "The Jury Stage" will be elaborated on here. He argues that The Jury Stage should focus on composing a jury consisting of both academics, politicians, civil society organizations, local residents, and people with practical knowledge about the erection of monuments, for example artists and architects (Young "The Stages of Memory" 37). With his criteria, Young underlines the significance of the democratic qualities of a memorial process, that is: working towards a multifaceted product and viewing debate and disagreement as an integral aspect of the process. Furthermore, Young insists on the fundamental importance of including *all* stakeholders at an early stage in the memorial process and continuing this collaboration and discussion throughout the rest of the process.

3.1 The Process in Amsterdam: *Het Nationaal Monument Slavernijverleden*

Although the inauguration of *Het Nationaal Monument Slavernijverleden* took place on 1 July 2002, the process behind its erection began almost a decade earlier with the foundation of the Nationaal 30 Juni/1 Juli Comité in 1993 (Kardux 171). This committee, consisting of descendants of enslaved people and people otherwise affected by Dutch colonialism, wished to increase knowledge about and visibility of Keti Koti in the Netherlands (Kardux 171). On 22 September 1998, the female descendant organization Stichting Sophiedela sent a petition to the Dutch ministry of internal affairs urging the Dutch government to get involved in official initiatives to commemorate the Dutch slavery past (Biekman). The petition, signed by Stichting Sophiedela's chair Barryl A. Biekman, did not specifically mention a slavery memorial or that the commemoration initiative should take place in Amsterdam, but it did start a debate about how to respond to the demands for commemoration put forward by descendant organizations (Biekman, Kardux 172). As a result, Het Comité van Aanbeveling Nationaal Monument Slavernijverleden [1] came into existence in 1999 and on 9 June 2000, the Dutch government and the Municipality of Amsterdam signed the *Convenant Inzake De Totstandkoming Van Het Nationaal Monument Slavernijverleden Tussen De Staat Der Nederlanden En De Gemeente Amsterdam*[2], hereon referred to as 'the covenant' (Boxtel). Thus, what started out as a descendant organizations-led project to foster recognition of the Dutch slavery past within the Netherlands eventually resulted in the erection of the first Dutch slavery memorial – a joint intiative between the Dutch state, the municipality of Amsterdam, Landelijk Platform Slavernijverleden[3] (LPS), and Het Comité van Aanbeveling Nationaal Monument Slavernijverleden (Boxtel).

The covenant of 2000 determined all major aspects of the to-be-erected *Het Nationaal Monument Slavernijverleden*, and particularly its 6^{th} and 7^{th} article show the main purpose of the memorial: to provide an easily accessible location for annual celebrations of Keti Koti, which appeals not only to descendants of enslaved people in the Netherlands, but to the Dutch society as a whole (Boxtel Artikel 6 and 7). The covenant further states that the decided-upon memorial must satisfy all involved parties – the state, the municipality, Het Comité van Aanbeveling Nationaal Monument Slavernijverleden, and LPS, an organization chaired by Biekman who represents the original civil society agents working on increasing focus on the Dutch slavery past and its implications in the present (Boxtel Artikel 7). An open call was made for design proposals for the memorial (Art at Site), and all parties eventually agreed on the design proposed by the Surinamese artist Erwin de Vries, which was then erected in Amsterdam's Oosterpark on 1 July 2002 (figure 1).

However, the actual realization of *Het Nationaal Monument Slavernijverleden* proved far less collaboration-oriented than what the covenant seemed to aim towards. According to Kardux, the government involvement in the decision-making process exceeded the covenant's initial consensus, since the Dutch state ended up dictating Oosterpark as the location of the memorial in spite of this being a far more peripheral part of Amsterdam than, for example, Dam Square, which some descendant organizations advocated for (Kardux 174). Kardux also argues that the Dutch state's intervention in the process caused the project to turn into a commemorative artwork for all Dutch minorities at the risk of removing focus from the intended purpose, namely to commemorate Dutch participation in slavery and counter a hegemonic Dutch narrative of slavery (Kardux 174). In addition to this, the circumstances of the official unveiling of the memorial brought discontent, as fences separated the memorial from the public in order to make it safe for the Dutch queen to attend the inauguration (Trouw "Protest"). This sparked protests from the citizens wishing to attend the inauguration, as they had to settle for watching it on screens (ibid.).

Although Young argues for the significance of disagreement and debate in the memorial process, he also writes that a modern memorial should "mesh memory with life, embed memory in life, and balance our need for memory with the present needs of the living" (Young "The Stages of Memory" 12). The Dutch state's interference with *Het Nationaal Monument Slavernijverleden*, both with regards to its placement and the significant broadening of who it should commemorate, thus appears at odds with the "needs of the living" (Young

[1] (In English) The Committee of Recommendation for National Monument for Slavery Past

[2] (In English) Covenant on the Establishment of the National Monument for Slavery Past between the State of the Netherlands and the Municipality of Amsterdam

[3] (In English) National Platform for Slavery Past

Figure 1: Photo of the Erwin de Vries' memorial inaugurated in Amsterdam in 2002 (Martin Alberts. Het Nationaal Monument Slavernijverleden. Stadsarchief Amsterdam, 9 October 2007, https://archief.amsterdam/beeldbank/detail/29fbdad7-ecfa-1ca1-7808-a8e013cb762e, accessed 1 June 2022).

"The Stages of Memory" 12), which one could in this case interpret as the descendant organizations who initiated the process. So, at the beginning of the memorial process, *Het Nationaal Monument Slavernijverleden* followed Young's success criteria, as it focused on including all stakeholders in the decision-making process, but the project eventually departed from Young's idea of a democratic and open memorial process because the Dutch state acted as an executive power by diminishing the wishes of other stakeholders, thus undermining the initially democratic nature of the memorial process. However, the functionality of the memorial as a site of commemoration and celebration of Keti Koti does reinstate its original purpose as a counter-hegemonic visual source specifically connecting to the memory of slavery in the Netherlands, and the fact that it remains the main site for Amsterdam's Keti Koti celebrations twenty years later (Keti Koti Festival) also shows that the redress purpose of the memorial succeeded in spite of state interference.

3.2 The Process in Middelburg: *Het Zeeuws Slavernijmonument*

According to the speeches given by GroenLinks municipal council member Jaap Goetheer and chair of the project group Zeeuws Slavernijverleden Roelof Koops at the official unveiling of *Het Zeeuws Slavernijmonument* in Middelburg, the process of erecting a slavery memorial in Middelburg began with the late 1990s discussions about where to place a national slavery memorial (Goetheer, Koops). Middelburg played a prominent historical role in the Dutch slave trade due to the slave trading activities of the Middelburgsche Commercie Compagnie (MCC) and West Indische Compagnie (WIC), which both had warehouses and other facilities in Middelburg in the $18^{th} - 19^{th}$ centuries (Campbell, Gelder).

When the 2000 covenant finalized the decision to make Amsterdam the location of the national slavery memorial, it sparked local initiatives in Zeeland, and also specifically in Middelburg, to erect a local memorial for the commemoration of Zeeland's involvement in the Dutch slave trade (Goetheer, Koops). On 31 July 2001, Ferdinand Ralf, the founder and chair of Stichting Monu-

Figure 2: Photo of the Middelburg memorial, designed by Hedi Bogaers (KITLV. Het Zeeuws Slavernijmonument. Flickr, https://www.flickr.com/photos/kitlvcollections/5612388631, accessed 1 June 2022).

ment Middelburg, proposed the idea to erect a local slavery memorial to the Municipality of Middelburg (Omroep "Middelburger Wil Slavernijmonument"). However, the municipality did not accept the proposal until Goetheer and GroenLinks Middelburg stated their support for the project on 9 March 2002 – a mere four months before the unveiling of *Het Nationaal Monument Slavernijverleden* in Amsterdam – and this time, the municipality approved the project (Omroep "GL Middelburg"). In 2003, Stichting Beeldende Kunst, in collaboration with the civil society organization Stichting Monument Middelburg and the Municipality of Middelburg, announced an open call for memorial designs and the three stakeholders agreed on the project submitted by the local artist Hedi Bogaers (Omroep "Kunstenaars Gezocht", Zierikzee).

As becomes evident from figures 1 and 2, the final design of *Het Zeeuws Slavernijmonument* differs quite remarkably from that of *Het Nationaal Monument Slavernijverleden*, since the Middelburg memorial is a stringent geometrical combination of three types of granite, whereas the Amsterdam one shows an evolution of human-resembling figures from slavery to freedom. However, the processes that led to the erections of the two memorials appear strikingly similar, especially when it comes to their inherent focus on collaboration between civil society and municipalities as well as the practical purpose of the memorials. For example, both erection processes aimed towards satisfying all parties involved with the realization of the memorials when it came to crucial decisions such as placement (Boxtel Artikel 6, Omroep "Locatie"). As explained previously, the final location of the Amsterdam memorial caused significant tensions between civil society organizations and the state/municipality. The placement of *Het Zeeuws Slavernijmonument*, on the other hand, seemed to satisfy all involved parties, since its erection in front of the MCC warehouse is in line with the history of this Middelburg building (Campbell, Gelder). *Het Zeeuws Slavernijmonument* was inaugurated on 2 July 2005, in order to not distort attention from the national Keti Koti celebration at *Het Nationaal Monument Slavernijverleden* on 1 July (Koops). But during his unveiling ceremony speech, Koops made it clear that in the future, he aspired to have the Middelburg memorial as the center of a more local Keti Koti celebration in Zeeland (Koops).

In contrast to *Het Nationaal Monument Slavernijverleden, Het Zeeuws Slavernijmonument* seems to have succeeded in continuing an open memorial pro-

cess, thus following Young's success criteria. Ferdinand Ralf, the initiator of the memorial process, initially faced a lack of support for his idea to erect a slavery memorial in Middelburg, but discussions between civil society and the Municipality of Middelburg eventually resulted in official support and funding for the monument. The design and placement of the monument constituted the result of the jury stage which included all stakeholders, and thus reflected Young's criteria for a successful, democratic memorial process more than the process in Amsterdam.

3.3 The Process in Rotterdam: *Slavernijmonument Rotterdam*

In 2009, a descendant group spearheaded by Rotterdam PvdA politician Peggy Wijntuin started collecting signatures from Rotterdam citizens in favour of erecting a local memorial to commemorate Rotterdam's crucial role in the Dutch triangular slave trade of the 18^{th} and 19^{th} centuries (BKOR "Slavernijmonument", Waterkant). Although *Het Nationaal Monument Slavernijverleden* already existed at that point, Wijntuin argued that a local memorial would make the inherent connection between contemporary Rotterdammers, the slaves, and the slave traders of the past more visible (Rijnmond). Wijntuin adds that the question of visibility makes it difficult for Rotterdammers to connect their city's history of slavery, because the triangular trading system meant that companies involved in the slave trade had warehouses and factories in Rotterdam. These produced the goods that the companies would sell in West Africa in exchange for enslaved people who would then get shipped off to the Dutch colonies (Rijnmond). According to Wijntuin, however, the historical lack of physical presence of enslaved people in Rotterdam, does not diminish the current issues faced by the city whose inhabitants do not connect Rotterdam's trade history and historical prosperity to the slave trade (Rijnmond).

As a result of the collection of 3,000 signatures through Wijntuin's campaign, the city's alderman Rik Grashoff from GroenLinks made an official promise on 23 November 2009 to erect a memorial (Waterkant). It would take four more years until the official inauguration, as involved parties wished to time the unveiling with the 150^{th} anniversary of the abolition of slavery in the Netherlands so that the memorial could exercise its function as a site for Keti Koti celebrations for the first time during this major anniversary (CBK). Since the erection of the Rotterdam memorial, the narrative of Keti Koti anniversaries in the Netherlands has changed, because of a more widespread acceptance of the fact that it took ten years from the official abolition of slavery in 1863 until enslaved people actually gained freedom in 1873, which results in Dutch cities such as Groningen currently planning to celebrate the 150^{th} anniversary of Keti Koti again in 2023 (Scheffer). However, in 2013 the memorial in Rotterdam still represented the 150^{th} anniversary.

Wijntuin collaborated with Het Centrum Beeldende Kunst Rotterdam (CBK), a public benefit organization funded by the Dutch state, in order to realize the slavery memorial. Together, they organized a design competition in 2012 where artists could submit ideas which representatives of Surinamese, Antillean, and Cape Verdean communities in Rotterdam would then vote on (BKOR "In Memoriam"). All stakeholders voted unanimously for the design proposed by Cape Verdean artist and Rotterdam resident Alex da Silva, whose *Slavernijmonument Rotterdam* according to the artist himself consists of an abstract interpretation of a slave trade ship made in bronze on top of which four metal figures dance their way from enslavement to freedom (BKOR "In Memoriam", CBK). However, the number of designs in the competition as well as further details about how exactly the vote was conducted is not known. According to BKOR, the competition was tough, but the article fails to mention exactly how many proposed designs this competition consisted of (BKOR). Da Silva, Wijntuin and CBK decided to place the memorial at the Lloydpier in Rotterdam, a location with significant importance to the Triangular trade since the ships set for West Africa would dock there (BKOR "Slavernijmonument").

Although some details regarding the memorial process in Rotterdam remain unclear, it still seems like a successful democratic process, especially since its Jury Stage included both civil society stakeholders, politicians, citizens, and an expert advisory group in the shape of CBK. Furthermore, this process made an attempt to further democratization of what Young calls The Judging Stage (Young "The Stages of Memory" 46), as it allowed minority communities in Rotterdam with a historical connection to Dutch slavery and colonialism to vote on a selection of designs chosen by the jury.

4 Contextualizing the Memorials within the Dutch Redress Debate

The process of erecting the Rotterdam memorial does in many ways reflect the processes in Amsterdam and Middelburg. *Slavernijmonument Rotterdam*, like *Het Nationaal Monument Slavernijverleden* and *Het Zeeuws Slavernijmonument*, originated as a civil society-led initiative seeking collaboration with a municipality in order to execute the slavery memorial. As with the two previous memorials, the Rotterdam process strove towards accommodating various opinions and the perspectives of minority groups within Rotterdam. Fundamental similarities between the three memorials also appear when looking at the intended function of *Slavernijmonument Rotterdam*, since the creation of a site for Keti Koti commemoration and celebration constitutes

Figure 3: Photo of the Rotterdam memorial, designed by Alex da Silva (NRC. Slavernijmonument Rotterdam. Nrc.nl, https://www.nrc.nl/nieuws/2018/10/05/monument-tegen-de-onwetendheid-a2139262, accessed 1 June 2022).

the main functional purpose of all three memorials.

In the autumn semester of 2021, I mapped racial redress activities in the Netherlands for the NIOD Institute of War, Holocaust and Genocide Studies' Transitional Justice Programme, and our research resulted in 59 mappings of apologies, statue removals, memorial erections, and other redress activities (NIOD data). This research illustrated three main tendencies within Dutch redress activities which will help place the erected and planned slavery memorials within a broader narrative of Dutch racial redress activities. The three trends are: first, clusters of commemoration around Keti Koti anniversaries and decennials, second, an internal process of change in-between Keti Koti anniversaries that highlights the effect of civil society organizations' pursuits of changing the hegemonic Dutch narrative of slavery and $17^{th} - 18^{th}$ century Dutch history, and third, activities sparked by world-wide Black Lives Matter protests of 2020. Of course, some redress activities overlap, but this does not defy the purpose of the categories, as this categorization illustrates general trends rather than mutually exclusive ones.

Firstly, 22% of the mapped redress activities either took their starting point on Keti Koti or function as preparation for bigger Keti Koti decennials – in particular those of 2013 and 2023 – which results in significant clusters of commemorative activities around Keti Koti (NIOD data). Therefore, the inauguration of the national memorial in Amsterdam on Keti Koti 2002, the deliberate one-day delay of the unveiling of the Middelburg memorial on 2 July 2005 to accommodate for the Keti Koti celebration in Amsterdam on 1 July, and the erection of the Rotterdam memorial for the 140^{th} anniversary of Keti Koti in 2013 all correspond to the broader trend for racial redress activities in the Netherlands connecting to Keti Koti. Furthermore, the Tilburg memorial inauguration will take place on 1 July 2022 and the three slavery memorials planned for erection in Utrecht, The Hague, and Groningen will be erected in 2023 to celebrate the 150^{th} anniversary of the abolition of slavery

in the Netherlands (Rubio, Scheffer, NOS, Hest). Both erected and planned slavery memorials thus follow the tendency for Dutch racial redress events to connect to Keti Koti anniversaries and even intensify this tendency by creating spaces for local Keti Koti celebrations, thus broadening the tradition for celebrating Keti Koti which previously mainly existed in Amsterdam (Ariese 125).

Although this research calls the second trend in Dutch redress activities 'an internal process of change', this process does of course still connect to global trends and changes in awareness of how to present unflattering elements of the national history of a country. However, the formulation of this second trend refers to the fact that during the late 2010s several museums and municipalities in the Netherlands initiated redress projects without being under a major pressure to do so. Some prime examples of this trend include the 2018 city council-initiated investigation of Rotterdam's slavery past and the formal apology from the Rotterdam major for the city's participation in the slave trade, the Mauritshuis Museum's decision in 2018 to remove a bust of Count Johan Maurits van Nassau-Siegen from its lobby due to his involvement in the Dutch slave trade, and the Amsterdam Museum's 2019 decision to cease to use the term 'Golden Age' in relation to Dutch 17^{th} century history (NIOD data). All these decisions show a potential shift within local municipalities' and museum's approaches to commemorating Dutch slavery history, since the city council of Rotterdam, the Mauritshuis Museum, and Amsterdam Museum all decided to engage in internal change without pressure from the public and protesters. These internal decisions to broaden or change the hegemonic narrative of Dutch slavery history potentially constitute long-term effects of the racial redress debate initiated by descendant-led civil society organizations in the 1990s and continued through racial redress activities and collaborations between civil society groups and municipalities during the 2000s and 2010s.

The three already erected slavery memorials and the ones planned in Tilburg and the Hague all follow the trend of internal change. As mentioned before, all erected memorials connect inherently to Keti Koti anniversaries, but at the same time they also include long processes during which a single member or a group from a minority community manages to establish the existence of public support or desire for a public site for slavery commemoration, thereby getting the state or local municipalities to engage in the process and fund the erection of memorials. Even though it may seem like the new wave of slavery memorials in 2022-2023 constitutes an example of attempts to diversify public spaces after the Black Lives Matter (BLM) demonstrations of 2020, two of the planned memorials actually follow the first trend rather than the post-BLM trend, as the official processes of erecting slavery memorials in Tilburg and the Hague began in 2018 and 2019, respectively (figure 4).

A staggering 47% of the mapped Dutch redress activities began after the BLM protests, which took its Dutch starting point on Dam Square in Amsterdam on 1 June 2020, and thus fall under the third trend detected in the redress data (NIOD data). This tendency corresponds with two of the memorials for slavery commemoration planned for erection in 2023, since the official support for these projects only arrived after the BLM protests of 2020, even though civil society agents and some politicians had lobbied for the memorials prior to the protests (figure 4; Lucas). The official municipal announcement of plans for erecting a slavery memorial in Utrecht happened on 9 July 2020 – a month after BLM protests began in Utrecht on 5 June 2020 (DUIC "Jaarbeursplein"). The BLM protests in Utrecht appear very fundamental to initiating the memorial process in Utrecht because the announcement came so swiftly after the protests began, and news articles reaffirmed this connection between BLM and the Utrecht memorial project by accompanying articles on the announcement of the memorial with pictures from a BLM-protest on Het Jaarbeursplein in Utrecht (NOS, DUIC "Slavernijmonument"). Groningen also experienced BLM protests in June 2020, which could explain why the official decision to erect a memorial came after 2020, even though the memorial process began before 2020 (Trouw "Demonstranten"). However, a connection between the memorial in Groningen and BLM appears less clear than the connection between official support for the Utrecht memorial and BLM protests in Utrecht.

In closing, although the BLM-protests had a major impact on Dutch racial redress activities it would be anachronistic to view the entirety of the new memorials planned for 2022-2023 as a consequence of BLM, since only two official announcements about memorial erections happened right after the beginning of BLM protest in the Netherlands. The new memorial processes and the three already erected memorials all follow the Dutch redress activity trend of connecting to Keti Koti, as the (scheduled) inauguration dates all systematically connect to Keti Koti anniversaries and decennials. Additionally, the erections of slavery memorials correspond to the trend of internal change detected within the racial redress mappings, since the majority of the erection processes try to place slavery within hegemonic narratives of Dutch history but do so through a process that includes both municipalities, civil society actors and local communities, thus contributing to a broader change in the Dutch narrative of colonialism and slavery history through the elaborate process required in order to erect slavery memorials.

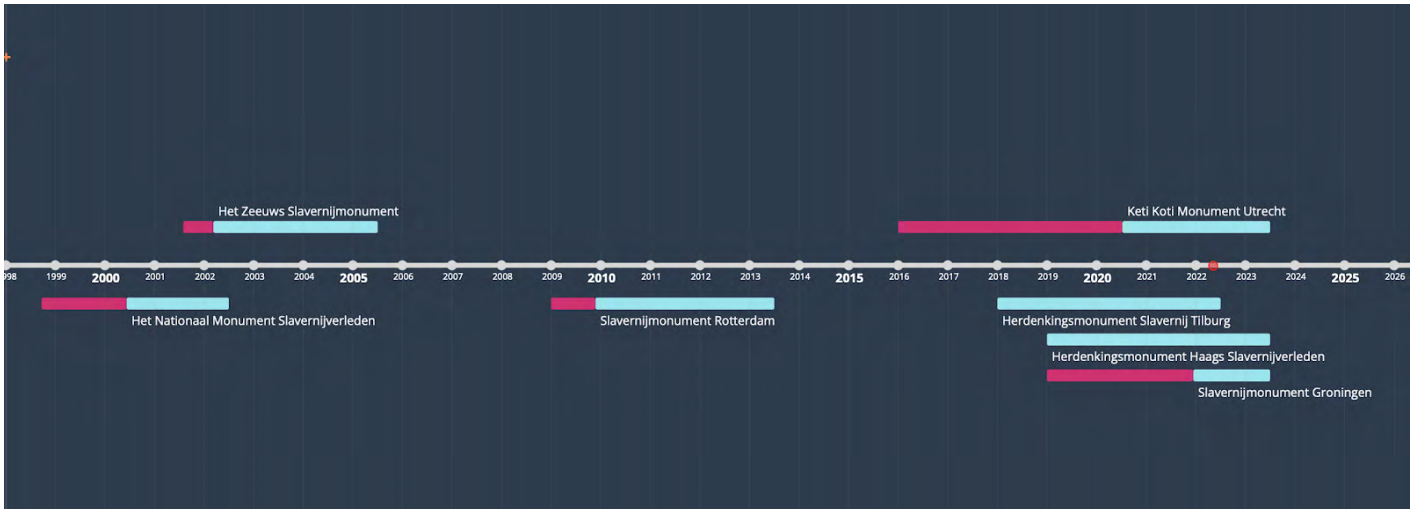

Figure 4: Timeline showing the processes of erecting slavery memorials in the Netherlands. The axis shows the years and the blue lines mark the timespan from an official municipal or state declaration about plans for erecting a memorial to the inauguration of the memorial. The red lines illustrate the amount of time that civil society organizations, often in collaboration with some city council members, lobbied for the erection of a memorial before it became an official municipal or state project. Sources used to create the timeline (from left to right): Boxtel, Kardux, Biekman; Goetheer, Koops; BKOR "Slavernijmonument, Waterkant, Rijnmond, Veenstra, Scheffer, Rozema.

4.1 Emplacing Erected Slavery Memorials Within Hegemonic Visual Narratives

The slavery memorials of Amsterdam, Middelburg, and Rotterdam all have quite distinct features in common with regards to the manner in which they occupy space: none of them have replaced existing monuments and all the erection processes did at least aim towards making the process of developing the memorials inclusive and democratic to accommodate for the people who would eventually use the memorials as places of commemoration and celebration. Thus, the three erected slavery memorials constitute examples of counter-hegemonic initiatives to visually commemorate slavery in the Netherlands that all revolve around the idea of placing something, which inherently connects to Dutch history, in the visual spaces which have thus far failed to provide public displays of commemoration of this element of Dutch history. Balkenhol's interpretation of "memory politics as processes of emplacement that intervene in a politics of belonging and provide a sense of political subjectivity" (Balkenhol 137) and Huyssen's argument that "histories of destruction, crime, and conflicts of all kinds" (Huyssen "Other Cities" 3) necessarily exist within visual spaces of cities provide the foundation of the following analysis and discussion of the three erected slavery memorials as counter-hegemonic visual sources emplaced in city spaces.

The connection between hegemonic visual spaces and emplacement becomes evident when examining the processes of determining where to place the three slavery memorials. The placements of the Rotterdam and Middelburg memorials signify attempts at emplacing slavery within city spaces that already fundamentally connect to aspects of the Dutch history and the slave trade, yet fail to represent this history through visual sources. Neither process had much difficulty arguing for the particular placement of a slavery memorial, since descendant organizations collaborated with the municipality of Middelburg in order to decide to locate the *Het Zeeuws Slavernijmonument* at the MCC warehouse (Omroep "Locatie") Also, Alex da Silva, the artist behind *Slavernijmonument Rotterdam*, wanted the memorial to abstractly represent a ship used for Triangular trade and therefore wished to place it at the Lloydpier (BKOR "Slavernijmonument"). The two memorials thus constitute examples of emplacements of the Dutch involvement in the slave trade into the visual source material of spaces which, until the memorial erections, only displayed the profit and prosperity of these two Dutch cities which originated from the cities' engagement in, for example, Triangular trading. Thus, the memorials occupy physical spaces, which already contain the aspect of Dutch slavery history that they represent without having visual source material to support this aspect of history, and emplace this aspect within the hegemonic visual narrative of the city space, thus providing a competing and broader visual narrative. The initiators of the memorials did not wish to remove the Lloydpier or the MCC warehouse, but instead aimed towards creating visual source material which supports a more nuanced hegemonic narrative of the Dutch slave trade.

Het Nationaal Monument Slavernijverleden in Amsterdam differs remarkably from the two other memori-

als, since its placement caused quite a dispute. Oosterpark, the location of the national slavery memorial, has no apparent connection to Dutch slave history and has only existed for around 100 years (Amsterdam Gemeente). According to Kardux, this location "was highly controversial in the Afro-Dutch community" due to its peripherality and lack of historical connection to the communities it supposedly represented (Kardux 174). To reiterate, the guidelines for the placement as described in the covenant stated that the location should be "accessible" and "representative", but did not further specify the criteria for a representative location (Boxtel Artikel 6). According to Kardux, when Boxtel and the Dutch state together with the municipality of Amsterdam became involved in the memorial process, the focus shifted from the original idea of commemorating the violence and consequences of the Dutch slave trade to the unity of all minorities within the Netherlands (Kardux 173). This shift in the focus of the memorial during the process of shaping and erecting could constitute a primary reason for its non-slavery-connected placement.

The process of erecting and placing *Het Nationaal Monument Slavernijverleden* distinguishes itself further from the erection processes for the memorials in Middelburg and Rotterdam by not being a project of emplacement. Because Oosterpark does not connect to the Dutch slavery past, placing the memorial there makes it impossible to constitute an example of emplacement of Dutch slavery history within a visual hegemony, since the memorial ended up merely being *placed* within a space it did not necessarily connect to instead of being *emplaced* within a space where a visual source of Dutch slavery history was missing. The municipality of Amsterdam's official website entry on Oosterpark further contributes to the notion that the location of the memorial focused more on broadening its meaning and making it less visible than on placing it in a historically/culturally significant space. The entry describes Oosterpark as a site for "fun festivals" and "a large paddling pool" but mentions neither *Het Nationaal Monument Slavernijverleden* nor the yearly Keti Koti commemoration/festival held at the memorial (Gemeente Amsterdam). The non-central location together with the apparent lack of connection between the memorial and its surroundings thus seem to strip it of the impact which slavery memorials can have on hegemonic spaces, as proved by the cases of Middelburg and Rotterdam. So, even though the Amsterdam memorial succeeds in providing a place for Keti Koti celebration and commemoration, it fails at intervening with hegemonic spaces.

4.2 The Current Wave of Local Slavery Memorials

The inaugurations of four new local slavery memorials will take place on Keti Koti 2022 and 2023 (Rubio, Scheffer, NOS, Hest), thus cementing the erection of slavery memorials as an actual trend for slavery commemoration in the Netherlands. Since only the Tilburg memorial currently has an official design and placement, and due to the scarce source material on the memorials planned for erection in Utrecht, the Hague, and Groningen, this chapter aims to explore the processes of erecting new local memorials through qualitative interviews with politicians, representatives of civil society groups, artists, and advisory organizations in order to gain these four different perspectives on the memorial erection processes. However, none of the approached people engaged with the process of erecting the monument in Groningen replied to the interview requests and the only response from a participant of the process in the Hague came in the shape of a brief email rather than a live interview, which means that this interview did not provide an opportunity for follow-up questions. All other qualitative interviews with stakeholders took their starting point in similar sets of questions relating to the processes of erecting local slavery memorials, but turned more dialogical by asking follow-up questions.

This chapter thus relies on the written response from Peggy Wijntuin, former PvdA-politician in Rotterdam, and current project leader of the memorial erection process in the Hague, a live interview with Leroy Lucas, initiator of Keti Koti Utrecht and the main civil society actor behind the memorial erection process in Utrecht, and live interviews with the artists behind the Tilburg memorial design, Albert Dedden and Paul Keizer, as well as Liesbeth Jans, process supervisor of the Tilburg memorial. Although the interviews do not provide a complete account of all groups involved, they constitute first-hand reports on the experiences of some of the agents behind the memorial erection processes, thereby making it possible to compare the processes to each other. This research fully acknowledges that other stakeholders involved in the process may have had different experiences of the processes, so all tentative conclusions made about the processes in Utrecht, The Hague, and Tilburg rely only on information presented by individual stakeholders, not the complete entity of stakeholders involved.

Wijntuin, a veteran when it comes to racial redress in the Netherlands, explains that the process of erecting a slavery memorial in the Hague differs from her previous experience with *Slavernijmonument Rotterdam*, since the city council initiated the memorial erection process in the Hague, whereas she initiated the process in Rotterdam herself as a citizen. Similarly, Jans states that the municipality of Tilburg contacted the advisory organization Kunstloc Brabant after the mayor

of Tilburg, Theo Weterings, promised the erection of a slavery memorial on Keti Koti 2018, which led to Jans' official involvement in the process from 2020 onwards (Taylor). Jans explains that at the beginning of this process, "there were two years of discussions between the municipality of Tilburg, various foundations, and the communities who will be represented through the monument", thus emphasizing the importance assigned to collaboration and debate during the first phase of the erection process. The processes in Tilburg and the Hague thus seem quite driven by the cities' respective municipalities from the beginning of the erection processes, something that differs from the heavily civil society-led initial phases of the memorials in Amsterdam, Middelburg, and Rotterdam.

To some degree, the processes in Utrecht and Groningen resemble the initial phases of the three already erected memorials for slavery commemoration, as it took civil society organizations years to receive official approval for slavery memorials in these two cities, but in both cities the collaboration with a few select members of the respective city councils provided a crucial addition to civil society campaigns for erecting slavery memorials. Although the municipality of Utrecht made an official declaration of its plans for erecting a slavery memorial in 2020, Lucas explains that the actual process began several years before that, and that the eventual official support only arrived after years of lobbying for the idea both within the municipality and through the popularization of Keti Koti in Utrecht:

> "The process started 6-7 years ago. (. . .) Melody Deldjou Fard [member of the city council of Utrecht for GroenLinks] told me that some people in the municipality were already talking about the idea of a monument, which I did not know at that point, so I told her that I would start working on the Keti Koti festival in Utrecht and she should start working on getting the monument approved by the Municipality – like a tandem project" (Lucas).

In Groningen, D66 politician Wieke Paulusma and the civil society organization Keti Koti Groningen approached the municipality in 2019 to inquire about the possibility for erecting a slavery memorial in 2022, but on 3 November 2021, an article by Sebastian Scheffer published in the local newspaper Oog stated that the initiators behind the monument had not heard any news about it since they proposed the idea of a memorial in 2019 (Veenstra, Scheffer). As a result, the initiators presented a new proposal to the municipality of Groningen on 14 December 2021 for the erection of a memorial in 2023, which was approved (Rozema 3). Due to the lack of responses to interview requests for the people involved in the erection process in Groningen, it has not been possible to gather updates on the process, and the rest of this chapter thus focuses on the memorial erection processes in the Hague, Tilburg, and Utrecht.

All interviewed parties state that the erection of local slavery memorials provides communities with a connection to Dutch slavery history and colonialism while at the same time focusing on unifying inhabitants of the Netherlands in the pursuit of a more equal future. Wijntuin mentions that the Dutch colonial past persists in present society through the 'Dutch cultural archive' – a term likely borrowed from the Afro-Surinamese Dutch social and cultural anthropologist Gloria Wekker's book *White Innocence* in which she uses Edward Said's term 'cultural archive' to show how a Dutch self-perception inherently connecting to the country's imperialist past remains strong among the white Dutch population today (Wekker 2). Wijntuin moves on to describe what she sees as the main purpose of erecting slavery memorials:

> "Our conviction is that mutual knowledge and insights about our shared history and in particular the colonial and slavery past contributes in the long term to both the prevention of mutual alienation and to our common Dutch identity and future". (Wijntuin)

The erection of slavery memorials does thus, according to Wijntuin, simultaneously challenge a hegemonic, white Dutch identity built on colonialism while still ignorant of the consequences of colonialism *and* create a new "common Dutch identity", which broadens the hegemonic and exclusive Dutch cultural archive in order to make room for an inclusive, shared interpretation of Dutch slavery history.

Although the interviewed parties share a consensus concerning the idealistic purpose of erecting slavery memorials, some interviews revealed more practical explanations for the timings of municipalities' official declarations to erect slavery memorials. According to Lucas, the BLM protests in Utrecht in June 2020 sparked a broader political interest in the memorial erection project in Utrecht:

> "Someone called up Melody Deldjou Fard after that [the BLM protests in Utrecht] and said that it might be time for that monument. I don't know who called her, but I think it was a white position. She knew she could pass a bill for a monument at that moment, because she had momentum. That's how it got done. But the bill had everything that I had been telling politicians for years in it, so I just saw my words, finally" (Lucas).

With the BLM protests, the municipality of Utrecht suddenly had a practical reason to support the erection of a local slavery memorial, even though Lucas, Fard, and others had tried to achieve this for years. The official collaboration between civil society and municipality thus began as a response to tumultuous protests rather

than as a result of dialogues between civil society, minority communities, and the municipality of Utrecht.

The Tilburg municipality is the second example of a municipality agreeing to erect a slavery memorial for practical as well as idealistic reasons. While explaining the reasons that the committee liked Dedden and Keizer's design, Jans revealed that its contrast to the existing and quite controversial Peerke Donders monument in Tilburg constituted an appealing factor:

> "Especially people from the BLM movement want it [the Peerke Donders statue] removed because it is a very stereotypical depiction of a relation between a white and a black man. That is one of the reasons why the Afro-Caribbean community in Tilburg wanted a memorial as a counterpart of the Peerke Donders monument. There is definitely a connection between the Donders monument debate and the erection of this slavery monument". (Jans)

According to Jans, the new memorial will help balance out the Peerke Donders monument and the hegemonic narrative of colonialism that it represents. This resonates with the Rotterdam and Middelburg memorials, as these also serve as examples of emplacing visual representations of counter-hegemonic narratives into hegemonic spaces previously dominated by visual sources that reinforce the hegemony.

Only Tilburg has settled on where to place its slavery memorial so far – a decision that caused debate between the municipality and the advisory committee for the memorial. Dedden and Keizer were not involved in this decision, but the artists do believe that the very central location assigned to the memorial signifies the serious recognition of what it represents. Jans explains that the municipality of Tilburg had not prepared for a situation in which the advisory board did not accept the proposal to place the memorial in the Vrijheidspark next to memorials commemorating the Second World War and the Holocaust. It therefore took time to finally settle on placing the memorial next to the stairs of Plan-T, a central location next to the station constantly frequented by students, travelers, and people on their way to work. Although this place has no connection to slavery, Dedden, Keizer, and Jans believe that the visibility of the memorial makes its placement significant, and this, in relation to the already erected slavery memorials, untraditional placement emplaces slavery history within a busy, contemporary visual space in Tilburg, thus attempting to continue the debate about colonialism, slavery, and how to create a more inclusive society.

4.3 Reflections on the Success of the Slavery Memorial Trend

In summary, several characteristics of the already erected slavery memorials reappear in the four new erection processes, among these the clear connection to Keti Koti, attempts at emplacing the memorials within hegemonic visual spaces, and creating memorials which focus both on broadening the hegemonic historical narrative of the Dutch colonial past and on encouraging hope for a more equal future. However, two of the four new memorial erection processes also show the difficulties of gaining official municipal support for slavery memorials, even after the completion of the erection processes in Amsterdam, Middelburg and Rotterdam. Although this chapter only bases itself on four interviews, the exploration of how key agents in the memorial erection processes have experienced the development from the initial idea, to create a memorial, to official municipal support for the project and, in the case of Tilburg, discussions about, and eventually decisions on the design and placement of the memorial, has provided a unique insight into the intricate processes of erecting local slavery memorials in the Netherlands.

To return to the question posed in chapter 2, this research shows that the Dutch racial redress trend of erecting slavery memorials does challenge existing hegemonic narratives, as the erection of a memorial generates debate between stakeholders, such as civil society organizations, municipalities, advisory organizations, and citizens in general. The Dutch state participated directly only in one memorial process – the process of erecting *Het Nationaal Monument Slavernijverleden* – and this example appears as the least successful memorial process according to Young's success criteria, since the state involvement led to the exclusion of other stakeholders initially integral part of the process and ended up dictating The Final Deliberation Stage, The Building Stage, and The Opening and Reception Stage. But even this least successful slavery memorial succeeded in creating a place in Amsterdam that still hosts annual Keti Koti celebrations twenty years after the inauguration of the memorial and constitutes the starting point of a successful wave of local racial redress initiatives working towards broadening the celebration, commemoration, and awareness of Keti Koti through the erections of local slavery memorials.

Although only three monuments have been erected and even though the first memorial erection happened quite recently in a historical perspective, the Dutch memorial processes seem highly successful as a means of racial redress and according to Young's success criteria. Particularly the process in Tilburg resonates with Young's perspective, as the memorial process according to Jans took longer than expected because civil society and advisory organizations did not accept the placement initially proposed by the municipality. This highlights that the stakeholders in the Tilburg memorial pro-

cess valued democratic decision making and debate over quickly finishing the project, which corresponds to Young's main argument, namely that one should view the memorial process as more important than the final memorial (Young 7).

However, the Dutch slavery memorials also differ from Young's main objective, since all the erected and planned monuments function as specific sites for Keti Koti celebration and commemoration. Covenants and other agreements about the main purposes of slavery memorials in addition to the interviews conducted with main stakeholders have all provided the same answer to questions regarding the reason for continuing to erect slavery memorials: the wish to create local sites for annual Keti Koti celebrations. All memorials besides the national one in Amsterdam constitute the products of *local* initiatives aiming to broaden the celebration and commemoration of Keti Koti by creating their own visual spaces. In that sense, in addition to the value assigned to the discussions and debates occurring during the memorial processes, the final, physical products of the memorial processes have a major significance too. Hence, because of local insistence on broadening and spreading Keti Koti beyond Amsterdam, the slavery memorials remain important even after their inaugurations due to their function as specific sites of commemoration and celebration every year on 1 July.

Even the most successful of the researched memorial processes face certain limitations and uncertainties. Young mentions community education as a vital step in the memorial process and a necessary addition to any project wishing to result in the erection of a physical memorial (Young 7). However, the local slavery memorial initiatives seem to focus far more on creating sites for Keti Koti commemoration than on combining the erection processes with educational projects. Jans even states that while the Municipality of Tilburg has supported the slavery memorial project, it does not wish to organize educational projects. This indicates a potential issue with erecting slavery memorials, as these memorials risk ending up as excuses for not engaging in additional memory work[4] on challenging hegemonic narratives of Dutch $17^{th} - 19^{th}$ century history. The Dutch government and the Municipality of Amsterdam actually created Het Nationaal Instituut Nederlands Slavernijverleden en Erfenis[5] (NiNsee) in connection to the inauguration of *Het Nationaal Monument Slavernijverleden*, thus ensuring the realization of a research institute in addition to the erection of a national slavery memorial (Kardux 174). However, Wekker argues that support for this educational project ceased after decreasing political support (Wekker 14), which again appears to support the suspicion that slavery memorials can constitute an excuse for the Dutch state and municipalities to not support further racial redress events seeking to provide counter-hegemonic narratives of Dutch slavery history and its continuing presence within Dutch society.

[4]Memory work: The manner in which a person, researcher or institution engages with the past in a historical, artistic and/or political manner

[5]The National Institute of Dutch Slavery History and Heritage

5 Conclusion

In conclusion, the Dutch trend of erecting monuments for the commemoration of slavery provides a highly successful means of racial redress, as it manages to emplace counter-hegemonic visual source material into spaces previously mainly occupied by visual sources furthering the hegemonic narrative of the Dutch participation in the slave trade. Furthermore, the processes of erecting memorials for slavery commemoration have resulted in many years of disputes and debates between minority communities, citizens, municipalities, and civil society organizations, thus furthering a public focus on what the slavery memorials represent and why certain groups seek to counter a hegemonic Dutch narrative of slavery through the erection of memorials. This aspect of the conducted research resonates with Young's insistence on the fundamental importance of the *process* rather than the *product* when it comes to modern memorial processes.

However, based on the information gathered about the intended purposes of all the memorials as sites for yearly Keti Koti commemoration and celebration, a main purpose emphasized in the qualitative interviews conducted with key members of the erection processes, is that Young's disregard for the physical memorial itself must be challenged in the case of the Dutch memorials for slavery commemoration, as the spread of Keti Koti celebrations and commemorations to cities other than Amsterdam connects to the memorial processes. So, even though these processes have a great significance when it comes to challenging a hegemonic narrative of Dutch slavery through debates and disputes between civil society organizations, citizens and politicians, the possibility to have a physical, fixed, and shared site for commemorating the abolition of slavery in the Netherlands also contributes to the significance of the monuments.

The process for a national slavery memorial constitutes the least successful one so far, as it went against Young's criteria for successful memorial processes because the Dutch state ended up dictating key decisions instead of allowing these to happen based on democratic discussions between all involved stakeholders. The Tilburg process, on the other hand, seems highly successful because it illustrates how a memorial becomes more valuable to a local community when all stakeholders can openly voice their opinions and challenge decisions proposed by the municipality, even though this prolonged the erection process. Generally, the other memorial processes appear to have learned from the mistakes of the Amsterdam process, as they seek to keep all stakeholders engaged through-

out the middle and final stages of the memorial processes instead of abandoning this democratic approach to memorial projects midway.

Future research on this topic would benefit from including interviews with multiple stakeholders per memorial from both old and current memorial processes in order to compare and contrast various stakeholders' experiences of the processes. Another important question unanswered by this research is how citizens interact with the slavery memorials outside of Keti Koti celebrations and whether the monuments manage to successfully counter a hegemonic narrative of Dutch slavery history then. Furthermore, although this research concludes that the slavery memorials pose a successful racial redress trend in the Netherlands, future research should look into whether politicians use the monuments as excuses for neglecting other redress projects, since this would provide an argument against uncritically continuing the trend of erecting local slavery memorials.

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Humanities

Painting Post-War Poland

Exploring Polish-Jewish History through Wilhelm Sasnal's *Such A Landscape*

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Abstract

This capstone focuses on Wilhelm Sasnal's exhibition entitled *Such a Landscape* from 2021 at the Museum of the History of the Polish Jews in Warsaw (Polin), which presents the artist's works engaging with the theme of Polish-Jewish history. I argue that Wilhelm Sasnal uses landscape as a tool to interrogate the traces of the Holocaust in Poland's everyday life, asserting the inner links between Polish identity and Holocaust history. This will be achieved through a formal and content analysis of selected paintings as well as the exhibition spaces, enriched by an analysis of the political, historical, and cultural context of Poland's relationship to the Holocaust. This study is significant as it offers a new perspective on Polish-Jewish history by understanding it through the lens of Sasnal's landscapes. Furthermore, this analysis enriches the ongoing debate on the Holocaust's position within Polish identity and offers new insight into a largely unprocessed strand of Polish history.

Keywords and phrases: *Wilhelm Sasnal, Holocaust, Polish-Jewish history, Landscape, Abstraction*

Suggested citation:
Miekus M. (2022). Painting Post-War Poland: Exploring Polish-Jewish History through Wilhelm Sasnal's *Such A Landscape AUC Undergraduate Journal of Liberal Arts and Sciences*, 19(1) pp. 126-145

imprint.

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1 Introduction

The Polish artist Wilhelm Sasnal (born 1972), attempts to work through what it means to be Polish by addressing the problematic aspects of his nation's history in his art. Sasnal calls himself a realist in that he depicts what already exists before him, whether that be his wife driving a car, food items, or politicians on TV ("Conversation"). Polish-Jewish history emerges as one of such themes that was always in Sasnal's field of vision. He says he finds himself drawn to Holocaust history. However, at times, "against his own will," (Sasnal, "Oprowadzanie") he continuously works with the remnants of this complex past that persists in translating itself into the present. Moreover, Sasnal undertakes the questions of Jewish history from a non-Jewish Pole's perspective. His art, therefore, invokes a tension between an insider and an outsider position. This part of his oeuvre was exhibited in 2021 at the Museum of the History of the Polish Jews in Warsaw (Polin) in an exhibition entitled *Such a Landscape* (2021). The show sparked considerable media attention, being Sasnal's first exhibition in Poland after 14 years. The artist has become almost a mythical figure in Polish culture with his success abroad (Szabłowski). This time he returned to his homeland with a widely acclaimed exhibition curated by the internationally recognized Adam Szymczyk[1]. *Such a Landscape* contains Sasnal's works from the last twenty years: some engage directly with the subject of the Holocaust, while others subtly play with the memory, associations, and mainstream visual culture connected to the topic. Sasnal takes a unique approach by focusing on Polish mental and physical landscapes and challenging their supposed neutrality. This capstone shows how Sasnal uses landscape as a tool to interrogate the traces of the Holocaust in Poland's everyday life, asserting the inner links between Polish identity and Holocaust history.

Further, as explained by Tokarska-Bakir, the Holocaust history "re-visits its changing audiences like a recurring dream even though the actors have long left the theatre" (3). Indeed, the tragedy inscribes itself on the present Polish landscape, whether through contemporary artists working within this theme or through recent political discourse aimed at safeguarding certain WWII narratives (Belavusau 37). Therefore, a need for a working-through of this strand of Polish-Jewish history echoes through Poland's societal landscape, and Sasnal, among other artists, undertakes this enterprise. The relevance of my research lies in assessing whether Sasnal's *Such a Landscape* exhibition impacts the discourse on the Holocaust in Poland. Does Sasnal offer a different perspective on a story told and re-told so many times? How does his exhibition fit into Poland's cultural landscape? Is it disruptive? This analysis is significant because it facilitates a discussion of the current relationship between Polish and Jewish WWII histories through a new perspective: Sasnal's landscapes.

Furthermore, the current crisis on the Polish-Belarussian border originating from rapid and uncoordinated migration adds another level of significance to Sasnal's exhibition. The artist acknowledges that because *Such a Landscape* engages in the theme of the 'other' within the Polish context, it can also be universalized to think about the situation on the border; specifically looking at the mechanisms that reveal the ongoing 'othering' of non-Poles ("Oprowadzanie"). Nevertheless, this study's primary objective is to analyze Sasnal's visual language to understand the intricacies between Polish identity and Jewish history.

2 Research Context

Poland's World War II history comprises a significant part of the nation's institutionalized heritage. Moreover, as highlighted by Sienkiewicz, Poland "is fond of dwelling on its war history and insurgent martyrology". Nevertheless, discussions on Jewish-Polish relations within the WWII framework are not met with the same enthusiasm in the realm of public discourse. Poland's relationship to the Holocaust remains an area of historical taboo as its traces "are perceived as damaging 'for the psychological equilibrium or identity' of the nation," (Tokarska-Bakir: van den Braembussche 6). While the process can be observed in Polish state institutions, the 1990s have seen a proliferation in arts and literature aiming to violate this taboo.

Most famously, Jan T Gross's book *Neighbors: The Destruction of the Jewish Community in Jedwabne, Poland*, (2000) has brought to light common practices of Poles wilfully committing crimes against their Jewish neighbors during WWII (133)[2]. The writer brings forward the story of Jedwabne, where half of the town's population killed its other Jewish half in atrocious crimes, such as trapping Jews in a barn and burning it down (20)[3]. In addition, the book moves away from the "perpetrators-victims-bystanders" axis, revealing its insufficiency in understanding the Holocaust history in Poland (12). Hannah Maischein, in "The Historicity of the Witness: The Polish Relationship to Jews and Germans in the Polish Memory Discourse of the Holocaust" (2018), also visualizes this complexity. She writes, "Poles could have been victims themselves, could murder or harm the Jews and therefore be considered perpetrators, and they could also help the Jews and therefore be remembered as heroes" (225). Moreover, Joanna Tokarska-Bakir analyzes the public reaction to Gross's book in her piece "Poland as the sick man of Europe? Jedwabne,

[1] Recipient of the Walter Hopps Award for Curatorial Achievement.

[2] Gross also writes about other pogroms carried out by Poles. Among them, he focuses on the Radziłów Pogrom of 1941 where a documented 800 Jews were killed by Polish "hooligans" (Gross 57).

[3] Gross quotes Szmul Wasersztajn's witness testimony outlining what happened in Jedwabne.

'Post-memory' and Historians" from 2003. She views the success of Gross's book as a symptom of an "epidemic of post-traumatic neurosis"(3): the book unexpectedly sold out and entered most fields of public discourse in Poland, thereby revealing the ongoing vitality of Holocaust history.

The relevance of the Holocaust history can also be observed in Poland's contemporary political scene. Uladzislau Belavusau, in "The Rise of Memory Laws in Poland" (2018), examines the phenomenon with attention to Poland's political climate. In 2018, the ruling nationalist right-wing Law and Justice party (PiS)[4] implemented a 'memory law' officially aimed at "countering misinformation" (Belavusau 42) about the Polish involvement in the Holocaust. Memory laws seek to regulate the different collective memories into a structured cultural memory that is in line with the current politics and self-image of the nation[5]. Belavesau argues that in the Polish context, the law is a guise for reverting to "populist identity-formation" by "mainstreaming nationalist historiography" (38). He ties this specific event to the broader context of memory laws in Europe that originate from Germany's implementation of criminal laws against Holocaust denial in the 1980s (38). Further, he indicates the years from 2010 onwards as a moment when many European countries implemented memory laws ranging in forms – "punitive and declarative, constitutional and administrative, legislative and judicial" – aimed at "counteracting historical disinformation" (38). PiS implemented such a law in 2018, illustrating the significance of the Holocaust debate to contemporary Poland's identity formation. The inclusion of such a law sponsors a specific history that dismisses any wrongdoings of Poles during WWII and instead commits to investigating violence perpetrated solely on Polish citizens, thereby supporting martyrological narratives (42).

Moreover, the memory law was met with backlash from Ukraine and the USA and deteriorated Polish Israeli relations (28). In response to the criticism, PiS minister Mateusz Morawiecki issued a statement explaining that the law aims to hold accountable those who spread misinformation about Poles' involvement in Nazi crimes and to condemn the denial of Poles' suffering under Nazi rule (Morawiecki qtd. in Belavusau 28). The phenomenon illustrates Poland's effort to safeguard its victim status and discredit accounts that undermine it, thereby further reinstating the taboo status to incidents like Jedwabne.

Nevertheless, the visual arts rarely reflect the state's position and have long been a locus for alternative representations of Polish-Jewish relations. The artists that have lived through WWII represent the Shoah differently than later generations. As argued by Feinstein, the 1990s marked a shift from sanctification to "deconstruction and innovation." The rules of depicting this history loosened, and the lines between what kind of representation is appropriate or not blurred. The artists belonging to this wave can be understood as what Marianne Hirsch describes as the "generation of postmemory" (347). They come after those who "witnessed cultural or collective trauma" (Hirsch 347) and remember what happened through existing accounts. This mechanism typically concerns the families marked by a traumatic past; however, Tokarska-Bakir shows how this memory transcends its immediate recipients. She calls it a phenomenon of displacement: "it occurs in a surrogate, symbolic place and moment, in a location removed in space and time from the events to which it refers" (4). Tokarska-Bakir further argues that this post-traumatic culture is not looking for healing, but is obsessively staring into its wound (4). Several Polish artists are products of this moment in time.

Figure 1: Zbigniew Libera, *Lego. Concentration Camp*, 1996, a box with legos, image from Contemporary Art Museum of Warsaw.

Zbigniew Libera is among the most famous Polish names working with the Holocaust history, known for his controversial (Feinstein) "Lego concentration camps" (see figure 1). The artist's pop-art objects comment on processes of commodification that reach over most aspects of reality. The piece can be read as a comment on the naturalization of antisemitism within mainstream culture as well as a visual provocation that tests the limits of representation. Libera's project represents a more polarizing approach towards Holocaust depictions, visualizing one way of engagement. Nevertheless, other Polish artists choose a more subtle method.

Joanna Rajkowska has been in the spotlight for creating Warsaw's first counter-monuments[6]. Her work *Oxygenator* is situated in the former Warsaw Ghetto territory, a space she describes as too powerful to confront

[4]PiS is notoriously known for populist policies such as imposing severe bans on abortion and for attempts at centralizing free media ("The Observer View").

[5]For an alternative definition of memory law see Belavusau and Gliszczyńska-Grabias 1-3.

[6]Natalia Krzyżanowska defines counter-monuments as "new tools of commemoration in the urban environment" (110) that "often [depart] from sculptural, ornamental or figurative imagery in favour of 'non-standard' artistic pursuits" (116). They bring "to the fore all the marginal and marginalised discourses of memory, and of a discursive construction of collective identity" (110).

(Rajkowska). Her artwork gives the public a chance to reclaim the space by situating a pond within the greenery that provides a breath of fresh air - both literally and symbolically (see figure 2)[7]. In the words of Rajkowska, "[I] wanted people to feel that they have the right to public space, that they can determine its fate, that they have a voice as citizens. I gave them the right, as it were, to Plac Grzybowski." The state sponsors a heavy commemoration of specific war narratives that marks places like these as untouchable. In contrast, Rajkowska makes the idea of healing and moving past traumatic histories viable. The two examples by Libera and Rajkowska hint at the complexity of artistic responses to the Holocaust in Poland, Sasnal's landscapes being one of them.

Figure 2: Joanna Rajkowska, *Oxygenator*, 2007, Grzybowski Square, Warsaw.

Sasnal's handling of the Holocaust history is unique as it dismisses Poland's efforts to separate the Holocaust from Polish history. Within that, it draws from Gross's enterprise in *Neighbors*. Gross writes that his book challenges "standard historiography of the Second World War,"(8) which splits its wartime history among two separate axes: "one pertaining to the Jews and the other to all the other citizens of a given European country subjected to Nazi rule" (8). He stresses the obsoleteness of this strategy in a Polish context, considering that Jewish people comprised one-third of Poland's urban population (8). He argues that Jewish history must be viewed as a central part of "Poland's modern history" (9). Sasnal's art carries out a similar objective by exposing its audience to traces of the Shoah in objects typical of every Polish landscape. The audience must then critically-assess their implication in this complex history.

Furthermore, Sasnal's interest in the landscape genre can be situated as a piece of a larger academic conversation. This capstone's analysis centers around understanding landscape as a tool that can reveal more significant societal and cultural phenomena. Simon Schama undertook this enterprise in his book *Landscape and Memory* (1995). Schama examines how landscapes construct and reflect cultural and national identities, thereby setting the framework for thinking about landscape as more than non-interfering nature. Another scholar working in this field, Martin Pollack, adds to the understanding of landscape specific to post-war Eastern Europe. His term "contaminated landscapes" that refers to sites of mass killings "perpetrated covertly" describes a large part of Poland's territory (qtd. in Tomczok 85). Sasnal depicts some of those areas in nuanced ways that aim to challenge the perceived innocence of landscapes.

Moreover, when situating Sasnal's works within the tradition of the Polish arts, one can see he is in conversation with the Romanticist visual language. In an interview with Banasiak, entitled "Przekleństwo Niepamięci," for *Szum* magazine (2021), Szymczyk expands on this argument, describing *Such a Landscape* as a progression from the heroic landscape genre typical of Polish Romanticism. He says he intended to "verify this great topos and to reduce it to something contemporary and specific."[8] Sasnal and Szymczyk are also interested in Poland's imaginary post-war landscape, often categorized in literature as empty, snow-covered, tragic, and beautiful (Szymczyk, "Przekleństwo"). The exhibition thus engages in a well-rounded analysis of what the Polish landscape came to represent. They invite the viewers to participate in this game of association and ambiguity, testing the limits of shared reading and understanding.

3 Methodology

The methodology of this capstone will include a formal analysis of selected oil paintings from Sasnal's *Such a Landscape* exhibition: *Kraków - Warszawa* (2006), *Chlew* (2011), *Kapusta* (2013), and *Pierwszy Stycznia* (2021). Any image I discuss in extent is included in the capstone. However, note that as information on dimensions was often missing, they were left out. Many of the canvases are around 55 x 70cm, while *Chlew* stands out; it is much larger, stretching out across the whole first wall of the exhibition. Further analysis will draw from Poland's political, historical, and cultural context to situate Sasnal's works within the environment from which they emerge and to examine the Holocaust's place, or lack thereof, within the Polish mental and physical landscape. I will utilize interviews given by the artist about this specific exhibition to enrich the discussion by bringing forward Sasnal's intentions with an awareness of his possible biases. I will additionally draw from an interview I conducted with Sasnal for this capstone entitled "Conversation". Adam Szymczyk was responsible for se-

[7] Rajkowska created *Oxygenator* in 2007 to commemorate the Warsaw ghetto (1940 - 1943) that at its peak held 450 000 people, the majority of whom were Jewish (Grabski).

[8] Translations are my own, unless otherwise noted.

lecting all the pieces in the exhibition. I have reached out to him but did not receive a response. I will thus consult existing interviews where Szymczyk discusses his curatorial choices to further understand the narrative being told by *Such a Landscape*. Moreover, the exhibition has been widely covered by the Polish media, allowing me to look at exhibition reviews among other secondary materials, including articles from major Polish newspapers such as *Gazeta Wyborcza* and *Zwierciadło* as well as smaller artistic magazines such as *Szum*. This capstone will also use an extensive list of online sources from Polin's website: recordings of debates about the exhibition's topics and guided tours provided by different experts. This approach will facilitate a well-rounded discussion of Sasnal's visual strategies and will help the viewer understand the landscape as a tool for revealing deeper social themes.

4 Chapter Outline

This capstone will be divided into three main chapters, each focusing on a different aspect of Sasnal's *Such a Landscape*. The first chapter will be dedicated to analyzing the exhibition itself. It will examine the space, the design, the specific context, and the curating strategies used by Adam Szymczyk. This section will aim to establish the narrative that is presented, and the means used to craft it. The consecutive chapter will then consist of a close reading of four selected paintings from the exhibition. These specific paintings were chosen because they are emblematic of Sasnal's varying approaches to depicting the post-war landscape. More precisely, *Kraków - Warszawa* and *Chlew* will visualize Sasnal's approach of mislabeling, while the analysis of *Kapusta* will expose how Sasnal disturbs the viewer by making the object familiar. Lastly, *Pierwszy Stycznia* will reveal the distancing approach used by the artist. Finally, the third chapter will connect the findings to their broader implications for Sasnal's art. Specifically, the treatment of abstraction and landscape will be assessed in more detail. This section will connect the arguments made throughout the paper and evaluate what they can add to the current discourse on the Holocaust history in Poland.

5 Chapter One: Situating *Such a Landscape*: Spatial and Historical Contexts

5.1 Polin: A Space for Remembering

In this chapter, I will present the history and context of the Polin museum to examine how it influenced Szymczyk's and Sasnal's approaches. I will then zoom in on *Such a Landscape's* exhibition space, assessing the effects created by the chosen design and curatorial choices. Creating a museum about the history of the Polish Jews was envisioned by the Jewish Historical Institute in 1995 and came into being in 2012. The museum is a public-private partnership institution formed together by the government, the local government of Warsaw, and the Jewish Historical Institute (Polin, "About"). The museum project has additionally received significant foreign funding from the USA, Great Britain, and Germany (Polin, "About").

The institution's name, Polin, originates from a legend about the arrival of Jews to the Polish lands in the Middle Ages. In Hebrew, "Polin" means Poland, whereas "PO lin" means "rest here" (Polin, "Legends"). The 15^{th}-century legend recounts that while the Jews entered Polish forests, they were greeted by birds chirping "PO lin" and saw a leaf from the Gemara hanging on the trees, and knew this would be their new home (Bader qtd. in Gliński). By using this legend, Polin expands its scope of interest beyond WWII history. The museum's mission is to deliver "1000 years of Jewish life on Polish lands," thereby moving away from solely focusing on martyrological narratives, and positioning itself as a museum that documents Jewish life rather than death (Polin, "About"; Polin, "Budynek"). Polin views itself as a space for dialogue where the past, present, and future of Jewish-Polish history can be understood and discussed. It aims to contribute "to the mutual understanding and respect among Poles and Jews" (Polin, "About").

Figure 3: Exterior of Polin, Warsaw.

Furthermore, the museum is situated in a space of cultural and historical complexity. The building stands on what used to be the territory of the Warsaw Ghetto that enclosed the Muranów district, stretching across 18km in length. This region was one of 600 ghettos located on Polish territory during WWII (Grabski). Moreover, Polin is located across from the *Monument to the Ghetto Heroes* from 1946, which pays tribute to the victims. The museum's architecture is in conversation with the emotional charge of the square. The building was designed by Finnish architect Rainer Mahlamäki whose concept won over the jury that looked at 100 different projects for the Polin building, including designs by

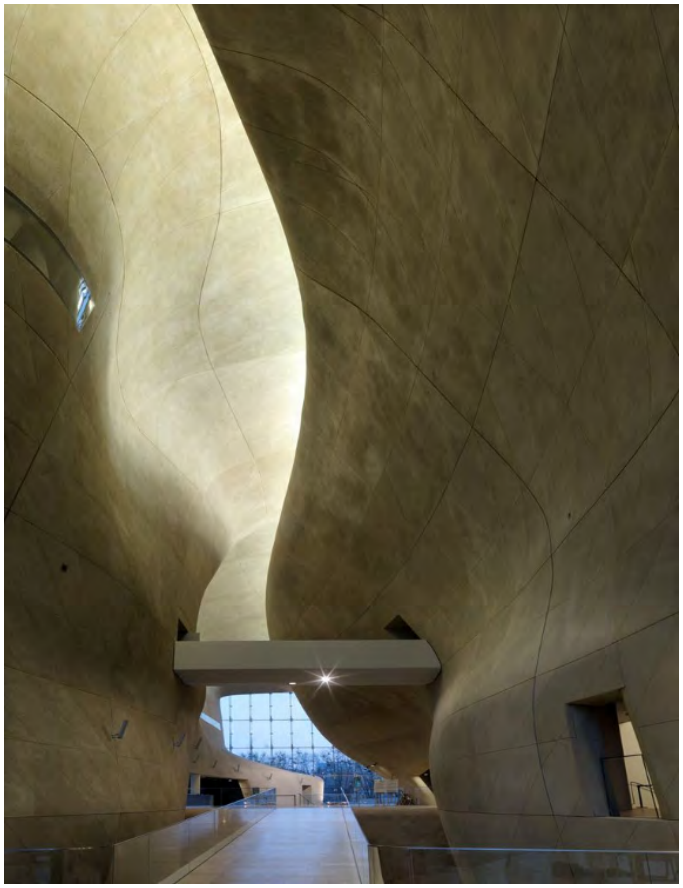

Figure 4: Interior of the large corridor entrance of Polin, Warsaw.

Daniel Libeskind (Polin, "Budynek"). Mahlamäki's concept speaks to Polin's mission of being a museum of life. The outside layer is a cube-like space covered in glass with inscriptions saying "Polin" (see figure 3).

The interior contains distorted walls that add dynamism and a feeling of flow to the space. These walls divide the museum among two axes and include a bridge that connects them, alluding to combining the past and the present (see figure 4). Windows cover the ceiling and walls to remind the spectators of the outside context (see figure 5). They add openness to the museum and remind the audience that the history being told here is not yet closed (Polin, "Budynek"). This design appropriately communicates the dynamism and the complexity of Jewish-Polish history.

Polin grants the space for temporary exhibitions to speak about different strands of Jewish history and culture. Their invitation of Sasnal is unique as it is Polishness rather than Jewishness that lies at the forefront of his art. Indeed, Sasnal is not mainly known for the part of his oeuvre that engages with Jewish history. Nevertheless, he has produced hundreds of paintings that speak to this topic (Szymczyk, "Przekleństwo"). What's more, in a debate with Szymczyk and Borchardt-Hume "Rozliczenie z Historią" (2021), Sasnal argues that Polin is perfect for displaying his paintings. He explains that, while he has avoided the term "political artist," he acknowledges that he becomes political by choosing to live in Poland and thus participating in political discussions and protests. When given a chance to say something through his art, he takes it ("Rozliczenie"). As a result, his art speaks to several political debates, although in subtle ways.

Figure 5: Interior of Polin, Warsaw.

5.2 *Such a Landscape*: Curation and Design

The context of the museum dictated Szymczyk's curatorial approach. He stresses that Polin is not a Holocaust museum, and thus *Such a Landscape* also refrains from focusing solely on this theme. He says that "these works are not only dealing with Jewish history but also addressing larger issues of hate of the others, xenophobia, and antisemitism today" ("Rozliczenie"). The selection includes 60 works from the span of 20 years of Sasnal's body of work which, together with the space's design, invite a nuanced viewing experience.

Szymczyk invited Johanna Meyer-Grohbrügge to design the interior of *Such a Landscape*. The visitors are led through a large room with small corridors that look like dead ends as they enter the exhibition. The white museum walls are covered in a thin tin layer for this show, allowing specific shapes and lights to be reflected (see figure 6). Sasnal admits he was worried about this exhibition design: "I was afraid this architecture was too present and too competitive for the paintings" ("Rozliczenie"). Nevertheless, Szymczyk was certain Meyer-Grohbrügge's design would intensify the viewing experience (Szymczyk "Rozliczenie"). The purpose of the tin material was to defamiliarize the viewers (Szymczyk "Rozliczenie"). When walking through these corridors, viewers see distorted reflections of their own body being forced to acknowledge their positionality within this space and this history. The walls also partially reflect other paintings in deformed ways, distorting the displayed content. The visitors are thus kept on their toes,

constantly carrying out plays of memory and association to decode the paintings. Further, all the corners are smoothed out, creating a gliding effect for the spectators. One flows from one room to the other without being confronted by any sharp turns or divisions. The space communicates that all these works, although different in form and content, belong to one whole. The space thus becomes a distinct landscape that leads you through the exhibition's narrative.

Figure 6: View of *Such A Landscape*, Polin, Warsaw.

Viewers must analyze the narrative of *Such a Landscape* in regard to Sasnal's and Sczymczyk's sensibilities. Both men explain their interest in landscape in the interview with Banasiak. They have personal stories that they view as moments emblematic of their critical re-evaluation of landscapes. These stories concern realizing that they live near Jewish graveyards - spaces of mass killings - whose existence has been normalized (Banasiak "Przekleństwo"). Szymczyk would walk by one such graveyard most days; it was right next to the supermarket, separated from the street by a stone wall. Sasnal strolled through a forest near his home and stumbled upon a space where hundreds of Jews had been killed. These stories inform the exhibition. Moreover, Szymczyk's approach also complicates the term "landscape." He plays with it by including paintings in the show that have nothing to do with this genre. The viewer's idea of what a landscape is becomes disturbed in the exhibition. Disruption of points of view and expectations, therefore, reappears as a prevalent theme in the exhibition.

5.3 The Artist's Voice

Moreover, a key characteristic of Sasnal's artworks is ambiguity. It is reflected in his hazy brushstrokes as much as in his politics. The artist strays away from revealing his political stance in direct ways. Instead, he opens up debates for viewers to engage with. One of the paintings that visualize this point is *Murzynek Bambo* from 2014 (see figure 7). The painting's title refers to a children's poem written by Julian Tuwim (1894-1953), a writer who occupies an important place in Poland's cultural memory. Tuwim's children's stories are read in most Polish households. *Murzynek Bambo* is one of such tales. In it, Tuwim invents the character of a young African boy who studies diligently at school, but when he returns home, he likes to frolic and cause his mom trouble. Tuwim himself, a Pole of Jewish descent, wrote the story in 1935 - a period when there were few people of color in Poland. Although the story is sympathetic towards its protagonist, it contains blatant racial stereotypes. Sasnal depicts this character in his 2014 painting, as indicated by the title. The piece contains abstract forms, floral motifs, and a small black figure standing towards the bottom of the canvas.

Figure 7: Wilhelm Sasnal, *Murzynek Bambo*, oil on canvas, enamel spray, 2014, Foksal Gallery Foundation, Warsaw.

One point of controversy has been surrounding Sasnal's choice to keep the original title. Art historian Małgorzata Baka, specializing in Sub Saharan art, took part in a debate entitled "The Other in Art" (2021), organized and hosted by Polin in conjunction with Sasnal's exhibition. She outlines the origin of the word "murzynek"

stemming from "Maurus," which means dirt, highlighting that the word always existed as a pejorative term. Nevertheless, this word has been widely naturalized due to its inclusion in casual vocabulary and literature, as exemplified by Tuwim's poem. Moreover, while there has been some growth in debates on race in Poland in the last few years, they do not yet translate into mainstream public beliefs. Sasnal, like many Poles, did not see the need to remove the phrase from his dictionary when he painted the work (Sasnal "Cztery Oczy"). However, the artist shares that after many debates with his son, he now understands the effort of moving away from such stereotypically charged phrases ("Cztery Oczy"). Despite this belief, he did not change the work's title for the exhibition.

Although controversial, the work reveals a more prominent characteristic of *Such a Landscape*. That is, asking the viewer to practice a critical gaze to evaluate Polishness through revisiting things that quietly exist in its cultural identity. What interests Sasnal is that Tuwim, who was himself an 'other' in Polish society, wrote about a character that was even more 'othered' within that context. Now, as this issue offers complex insight into the practice of imposing hierarchies of people perceived as foreign to a particular nation or society, it also reinforces problematic assumptions. Sasnal said that he does not think Tuwim was trying to be racist, as he was Jewish and thus was himself an outsider ("Cztery Oczy"). Sasnal is thus led by the assumption that Tuwim's ethnicity clears him from having racist intentions when writing the story. This naive belief is a fallacy: anyone can be racist regardless of ethnicity. While Sasnal's sympathy towards Tuwim can be symptomatic of his environment where antisemitism is prevalent, we must critically assess his approach. Sasnal redeems Tuwim due to his outsider position, but *Bambo's* depiction remains a dark unindividualized figure that represents a stereotype but does little to dismantle it. In that sense, the viewer is left with ambiguity when trying to decipher the politics behind this painting. Nevertheless, the inclusion of *Bambo* in *Such a Landscape* is significant in exposing complicated strands of Polish culture. By removing the image of *Bambo* from its usual context - a children's poem book - Sasnal invites the audience to critically assess it, undermining its neutrality.

It is, therefore, evident that *Such a Landscape* does not purely focus on Holocaust history. The point of contact between all the works is Sasnal's interrogation of Poland's cultural and historical landscape. Ruchel-Stockmans illustrates this strategy well. She writes that Sasnal is "attracted to themes and figures from his country's past because he wants to know what it means to be Polish and at the same time needs to disengage himself from that legacy and identity" (218). Entering into conversation with materials silently existing within Polish culture and making them yet again visible through his art is one of Sasnal's forms of engagement with his Polishness.

6 Chapter Two: Removing and Exposing: Analyzing Sasnal's Landscapes

How does one depict a landscape that has witnessed war and horror? Schama describes Poland as a "haunted land where greatcoat buttons from six generations of fallen soldiers can be discovered lying amidst the woodland ferns" (24). It is a place with, "raw, chafing histories torn from decades of official silence yet still imperfectly recovered; markers freshly dug or posted" (25). Sasnal engages with precisely these complex historical elements within his works; he reads through them and faces them, yet never in direct ways. Indeed, the Polish-Jewish history that Sasnal addresses in his art is heavy and dark, and he, therefore, sets strict boundaries for ways in which he can depict it. He views direct and bare representations of the Holocaust landscape as "perverse and banal" and thus always places distance between himself and this history (Sasnal, "Cztery Oczy"). This distance is created by placing objects between him and the landscape and employing abstraction. The discussion that follows will expose the artistic choices that keep Sasnal, "protected from the landscape" and that keep "the landscape protected from [him]" (Sasnal, "Cztery Oczy").

6.1 *Kraków - Warszawa* and *Chlew*: Mislabeling

Kraków - Warszawa (2006) and *Chlew* (2011) are emblematic of Sasnal's mislabeling approach. *Kraków - Warszawa* depicts a field with a single figure, an electrical tower, and a cloud of smoke (see figure 8). The oil paint exhibits expressive strokes to construct horizontal and diagonal lines that make up the green field. There are different shades of green with a white foreground peeking from underneath. The field takes up most of the canvas and transitions into a gray sky. The centrally positioned cloud of smoke emerges as the focal point of the painting. The cloud is dense in paint and brushstrokes, making it appear dynamic; one can perceive its motion through the wind. The figure resembles a human shape but maintains anonymity. We do not see an individual.

Chlew is a long, rectangular canvas that stretches along the first wall of the exhibition (see figure 9). The oil painting displays two large industrial buildings with white walls, gray roofs, and small black windows. The two structures are long and flat, unmarked, and lacking any individualizing characteristics. They look empty and dull, creating a cold and eerie atmosphere around them. A bright green field surrounds the buildings with some black vegetation and a muted blue sky. The grass field is similar to an ocean with its wave-like brushstrokes and hints of lighter color. The black bushes around the buildings add a disturbing feel to the painting; there is dark

paint spilling out from under them, reminiscing flows of blood.

Figure 8: Wilhelm Sasnal, *Kraków – Warszawa*, 2006, oil on canvas, Zachęta collection, Warsaw.

Figure 9: Wilhelm Sasnal, *Chlew*, 2011, oil on canvas, Foksal Gallery Foundation, Warsaw.

Schama writes that, "we are accustomed to think of the Holocaust as having no landscape - or at best one emptied of features and color (. . .)" (26). When the Holocaust does not have a recognizable and stable landscape in our minds, it can appear and reappear in many different places. Sasnal shows that the perception of the Holocaust landscape rests on specific symbols that our minds associate with this strand of history. The two different settings depicted in these paintings have no direct connotation with the Shoah. *Kraków - Warszawa* and *Chlew* represents Sasnal's train journey from Kraków to Warsaw, during which he took a picture of the view exhibited in the painting. The view depicted is a figure burning some old leaves, but the viewer's mind infers different meanings when first glancing at the image. Similarly, *Chlew* portrays an industrial pork slaughterhouse, yet is reminiscent of concentration camps' barracks. Sasnal, therefore, invites the audience to participate in a play of mislabeling that exposes their expectations of the exhibition.

What I call 'Sasnal's mislabeling approach' is a process that occurs in the viewer as an effect of Sasnal's work. Sasnal makes space for the audience to project their expectations onto the canvas, causing a mislabeling of the depicted objects. The artist invites this process as it exposes and contradicts the viewer's preconceived assumptions. One enters the museum of the history of the Polish Jews and sees *Chlew* as the first painting of Such a Landscape. Immediately the viewer begins inferring meanings. The two slaughterhouses look like barracks, and the dense smoke from *Kraków - Warszawa* and *Chlew* echoes the smoke from the gas chambers. Nevertheless, Sasnal deceives the audience. Indeed, it would be against his artistic sensibility to depict actual barracks and smoke clouds from the concentration camps. Instead, one sees everyday occurrences that gain new meanings when placed in this specific context. Sasnal thus makes the audience hyper-aware of their surroundings and how those affect their readings of his works. The setting, the Museum of the History of the Polish Jews, makes the viewers search for symbols and markers of the Holocaust. Sasnal plays with these assumptions by inviting the audience to see these markers in his paintings.

The most potent effect of Sasnal's mislabeling approach is that it reveals more about the audience than about the artist: the viewer does the mislabeling. By including these two artworks in the Polin exhibition, which dives into post-war Poland's mental and physical landscapes, Sasnal and Szymczyk dissect the audience's expectation of the Holocaust landscape. This landscape lacks constant physical representation but disappears and reappears in the viewer's mind throughout the exhibition. It is engraved in the Polish collective memory in the form of particular images. Sasnal plays with these expected visuals to unsettle the viewer's preconceptions. What is revealed is that the Holocaust landscape is nowhere and thus simultaneously is everywhere. One sees it in the smoke clouds, industrial buildings, and everyday settings. Sasnal's works invoke a paranoia captured by the re-emerging question: Is what I am seeing what I think it is? One suddenly sees the Polish-Jewish history lurking in every painting. The result of this enterprise is exposure to the Shoah through Poland's everyday landscapes, thereby revealing they are not innocent.

6.2 *Kapusta*: Disturbing by Making Familiar

Kapusta (2013) is a medium-sized canvas depicting cabbage heads (see figure 10). The cabbage heads are stacked on top of each other and form one large pile. They are alike, however, they have some individualized traits. The insides of the cabbages are painted in light green shades with darker green contours, some more persistent than others. Sasnal adds depth to the cab-

bages by using visible brushstrokes in shades of green with white shading in the centers. Their shapes and colors are imprecise, with some of the heads blending into each other and others having some loose paint strokes. The depiction keeps its realist character in the figurative treatment of cabbage but is painted with a degree of abstraction. The background emerges from behind the vegetable pile and suggests a dark sky. This sky contains hues of grays and blues, and has some white strokes peeking from under the dark clouds. There is a dark shadow behind the pile, which dramatically brings out the color contrast between its dark hue and the light cabbages.

Figure 10: Wilhelm Sasnal, *Kapusta*, 2013, oil on canvas, Foksal Gallery Foundation, Warsaw.

The story of cabbage coming to Poland is one of the country's favorite myths that many children are taught in primary school. The tale goes that in the sixteenth century, the beautiful queen Bona Sforza, the Italian wife of Polish king Zygmunt the Old, had brought with her to Poland all kinds of wonderful vegetables, among those cabbages (Kucia). The vegetable has a sacred place in Polish history. It is also a very cheap and common vegetable found in most Polish households. Specifically, it is widely harvested in the countryside and fermented to become a staple food eaten all year round. It also has a prominent place on the Christmas table: it is used for "pierogi" (Polish dumpling), bread stuffing, and a "bigos" stew. It is also the main ingredient of the most traditional Polish soup, "kapuśniak" (cabbage soup), and a staple meat dish "gołąbki," which translates into "pigeon" but is actually pork wrapped in cabbage. Cabbage is also associated with life in communist Poland. It was one of the few readily available vegetables and was the star of the most typical dishes from that period, such as "łazanki" (small square pasta). Cabbage is thus embedded into Polish culture simultaneously as a symbol of poverty under the Polish People's Republic [9] as well as prosperity in rural Poland (Tymchowicz). Its cultural symbolism and importance in Polish cuisine thus foster its status as a symbol of Polishness. Within that, Sasnal depicts a typically Polish theme in his painting and disturbs its meaning.

Szymczyk discusses how the paintings included in Polin display a "withdrawal of content" ("Rozliczenie"): what you see is blurred, and what you know appears foreign. Indeed, the pile of cabbage is a familiar image, typical of walking past most street vegetable stands. However, Sasnal's depiction robs the vegetable of its familiarity, inviting new readings to be drawn by the audience. The round objects no longer suggest cabbages but human heads piled on top of each other. The depiction references the imagery of piles of items found in Holocaust museums.

Figure 11: View of the shoe piles in the Auschwitz-Birkenau Museum, Oświęcim.

Poland's Auschwitz-Birkenau Museum displays massive shoe piles behind a glass wall and a pile of hair of the concentration camp's prisoners (see figure 11). Sasnal shares that in his city, children went to see the Auschwitz Museum at the age of 13 ("Conversation"). He considers this age too young to be confronted with such imagery. The impact of the piles is straightforward and disturbing; I remember being horrified by these rooms when I first came to Auschwitz as part of a school trip. The pile imagery, thus, comes to haunt the viewer when facing *Kapusta*. The everyday landscape quickly acquires a dark and twisted connotation. The cabbage ceases to be a simple vegetable and is transformed to remind the audience of the victims of the Holocaust. Sasnal shows that even the most everyday objects have this history inscribed in them. The paranoia returns,

[9] Polish People's Republic was the official name of Poland in the years 1952 to 1989 when it was a communist state under USSR's political control.

forcing the viewer to rethink what appears before their eyes.

Furthermore, Sasnal's *Kapusta* plays with Polish imagery to assert a connection between Polish identity and Holocaust history. The state-sponsored and institutionalized WWII history is categorized by narratives of heroes and bravery rather than by the nation's implication in Nazi crimes. As put by Gross, "the memory, indeed the symbolism, of collective, national martyrology during the Second World War is paramount for the self-understanding of Polish society in the twentieth century" (143). By connecting the typical Polish vegetable with the victims of the Holocaust, Sasnal is questioning the state-sponsored martyrological discourse of innocence around WWII. Poland's effort to safeguard its victim-status is further exemplified by the debate sparked by Gross's book *Neighbors*. Tokarksa-Bakir analyzes the public reactions to the book and systemizes the various responses into specific denial mechanisms. She views this, "mythological-type suppression,"(6) as the most prominent denial mechanism that attempted to segregate the perpetrators of Jedwabne's crimes into social outcasts. She shows how this attitude allows the nation to uphold its mythical moral purity. The implication is, "that whoever commits such a crime becomes an outcast by definition, and thus the nation that has issued him remains untarnished" (Tokarska-Bakir 6). The spokespeople of this denial mechanism imply that it was not ordinary Poles who murdered the Jews but rather troubled hoodlums. This strategy inscribes itself into the broader effort of the Polish government to separate Holocaust history from Polish history. Sasnal's painting complicates this effort.

In the painting, the Polish symbol of the cabbage is inherently connected to the dead bodies of Holocaust victims. The result is powerful in that the audience starts mistrusting everyday objects and begins scrutinizing Polish-Jewish history through them. The Polish cabbage loses its innocence, simultaneously causing a fracture in Poland's self-image relating to its Holocaust history. Moreover, the Jewish victims of the Holocaust were, in many cases, fully assimilated Poles; nevertheless, the post-war narratives tend to minimize their belonging to Polish society. In that sense, Sasnal's use of a very Polish object to allude to Holocaust victims can be read as the artist's attempt at safeguarding the Jews' position within Polish culture. Sasnal asserts the internal links between Poland's post-war identity and the Holocaust history by using these visual strategies. The artist refuses to separate the two histories into foreign strands, thereby disrupting the institutionalized WWII history's portrayal.

6.3 *Pierwszy Stycznia*: Distancing

Pierwszy Stycznia (2021) is one of the two most direct representations of the Holocaust in the exhibition, as it shows the gate of Auschwitz-Birkenau (see figures 12 and 13). The piece is a diptych based on a photograph Sasnal took while he and his wife, Anka, drove back from a New Year's Eve party in January 2021. The artist chose to take the route that passes by the Birkenau gate. In the interview, Sasnal told me he was shocked that he could stop his car on top of the train tracks that were used by Nazis to transport prisoners during WWII ("Conversation"). He felt the need to paint this experience ("Conversation").

Figure 12: Wilhelm Sasnal, *Pierwszy Stycznia* (side), 2021, oil on canvas, Foksal Gallery Foundation, Warsaw.

Figure 13: Wilhelm Sasnal, *Pierwszy Stycznia* (back), 2021, oil on canvas, Foksal Gallery Foundation, Warsaw.

Anka's face emerges in the foreground of the canvas. Her features are treated with detail: a shadow falls on the side of her face, and there are light spots on her eyelids, chin, and forehead. Anka's frowned eyebrows communicate a sense of focus or worry. In the

side painting, she is looking towards the front, and in the back image, she is looking out the window towards the Birkenau gate. One can see Sasnal's characteristic meandering brushstrokes in the depiction of her hair.

The inside of the vehicle takes up the second layer of space within the painting. It is treated with a similar level of realism. There is light entering the car from the windows, which appear to be completely transparent, and they open into an abstract external world. The dark colors from the inside of the car contrast with the light hues of the outside view. The treatment of the gate is symmetrical and geometric but at the same time imprecise and blurred. The train track reaches from the couple's car until the opening of the gate; it is placed in the middle of the painting, inviting the viewer to follow it with their gaze.

While the car's interior is depicted quite realistically, the outside world is constructed with abstraction. The background is entirely white, and the gate and parts of the ground are painted in a shade of pink, with light green grass peeking through from the side. The structure appears to have no weight to it: an effect created by the lack of detail and light hues that merge into the ground. Thus, while Sasnal paints the Birkenau gate, he refuses to depict the structure accurately and directly.

Pierwszy Stycznia is almost an inverse of the processes at work in *Chlew*, *Kraków - Warszawa*, and *Kapusta*. While in the previous paintings, I have shown how Sasnal takes a neutral site and charges it, here he takes a charged site and neutralizes it. The gate's representation does not spark the horror it stands for. It is rather subdued to the point of loss of its identity. For Sasnal, painting is a "space for trial: what can be represented?" ("Cztery Oczy"). Here, he concluded that he could not directly represent the site's charged history ("Conversation").

Hannah Maischein writes that,

> "(...) witnesses are media themselves because they transmit what they have seen. Thus, they seem to allow a very auratic relationship to the historical event for others who come in contact with them: the trace of the event is inscribed in the witness' memory like the light on a photograph" (Maischein 216).

Maischein's argument can be expanded to think about landscape as a witness. Indeed, the Holocaust is inscribed in Birkenau's identity to the point of total domination of its history. Auschwitz acts as a space crucial to the cultural memory of European Jews and attracts millions of tourists yearly ("Aktualności"). For Poland, the place occupies a problematic space within the country's national identity. Its striking traumatic charge alienates the site from its surroundings; Auschwitz becomes an almost alien figure in Polish history used to tell stories of individuals, some of whom had little in common with this country. Indeed, there is a visible process of disavowal of these lands illustrated by the 2018 memory law discussed above. It condemns the discursive linking of Poland and the Holocaust through terms like "Polish concentration camps" (Belavesau). Thus, spaces of Nazi crimes are not seen as belonging to Polish towns but rather as foreign objects that happen to be there.

Nevertheless, Birkenau used to be like every other Polish town. In 1941 most of the citizens of Birkenau got forcibly reallocated to make space for the camp construction ("Mieszkańcy"). Some of the town's inhabitants came back to what was their home after the war. Today, Birkenau hosts around 2,500 people living around the remnants of the concentration camp ("Wieś Brzezinka").

Sasnal's *Pierwszy Stycznia* is in conversation with the history of Auschwitz-Birkenau. The interaction between the landscape and the drivers is simultaneously casual and extremely tense. The couple is on their way back from a party and passes Birkenau on their way home: this view is someone's everyday reality. It is not hidden away in a corner somewhere, but it merges with the roads of the Wisła region. On the other hand, there is an undeniable distance between the couple and the gate. Sasnal says, when asked about this painting, that he "always need[s] to have a prop that will cover me, distance me, from this landscape" ("Przekleństwo"). Here, the car and Anka act as such elements that put a literal and figurative distance between Sasnal and the gate. By depicting the image from within the vehicle, "the gate is not exposed to our presence which can be vulgar for both sides," says Sasnal ("Conversation"). To him, painting the direct realist image of Auschwitz would be a perversion (Sasnal, "Cztery Oczy"). He thus undertakes several strategies to distance himself from the landscape.

Anka acts as the most visible shield for the artist. She is closer to the view, Sasnal is behind her. He pays a lot of detail to her depiction, emphasizing her as an essential part of the artwork. Moreover, a perceived closeness to her figure creates a feeling of familiarity and protection. In that sense, the painting is a site of an interplay between intimacy and distance. Anka represents the former, while the Birkenau gate represents the latter. Sasnal uses this strategy to communicate the complexity of viewing Auschwitz in a casual setting. Bypassers are exposed to this view and the impossibility of grasping the crimes it stands for. The crimes deprive the place of its familiarity and its status of just another Polish town. Anka is thus a strong contrast to this setting.

The car is another tool that distances Sasnal and the audience from the landscape. On the one hand, it has a similar role to Anka in that it is a shield of familiarity for Sasnal. On the other hand, the materials that appear in the car have another significance. The window separates the couple from the Birkenau gate. It is through the see-through glass that we glance at this landscape. Sasnal depicted the window to be completely transparent, making it seemingly connected to the view while at

the same time using it as a point of division. Moreover, looking through the window is an act of spectatorship. One can compare it to watching a film. It becomes easy to become a passive spectator separated from the image by being inside of the comfortable car. Yet, Sasnal took a photo of the view and painted it, thereby actively implicating himself in creating this landscape.

Finally, another critical element of the vehicle is the side mirror. Sasnal says that he leaves the meaning behind this element open to the audience ("Cztery Oczy"). While the mirror reflects what is already behind the car, it can invoke intentions of looking into the past. The modern interior of the car reminds the viewer that we are in the present, but the past is following us. This reading connects the dots between then and now to show that this history is imprinted on the present. Tokarska-Bakir refers to this phenomenon when stating the following question about the Shoah, "[w]hat is the mystery buried in the sixty-year-old tragedy that refuses to be dismissed (...)?"(3). Indeed, the Holocaust history comes back to haunt the generations who come after, among many ways, in the form of the contaminated landscapes. Sasnal, like many Poles, keeps staring into this past but constantly interrogates it through the present context.

7 Chapter Three: The Effects of Sasnal's Visual Language

7.1 Abstraction and Realism

This section will consider the broader reasoning and effect behind Sasnal's use of abstraction and will consecutively lead into a discussion of Sasnal's use of landscape. Abstract forms reappear throughout the exhibition in different artworks. They are often invoked by Sasnal's use of thick, expressive brushstrokes that stray away from figurative shapes. The viewer follows these strokes on a quest to decode the meaning behind the paintings and often discovers atrocities when piecing the components together. Further, as outlined by Redzisz, abstraction is felt in Sasnal's works where, "objects [are] isolated from their original contexts, or enlarged to the point of losing their identity." The objects are dramatized to distort their realist character and removed from their usual setting to denaturalize their innocence. Sasnal consciously meanders between styles to create specific effects. Hauser & Wirth's exhibition catalog states that Sasnal, "navigate[s] between figuration and abstraction, eschewing a definitive narrative or agenda". Indeed, such abstraction allows Sasnal to maintain a level of ambiguity, which is perhaps necessary when depicting histories that are so heavily charged.

7.2 Depicting the Undepictable

More generally, abstraction is one way to represent the Shoah visually.[10] Mark Godfrey explains the line of thought favoring this approach in *Abstraction and the Holocaust* (2007). He writes, "abstraction, as a non-representational art, is the most appropriate kind of art to respond to an event that is beyond representation" (6).[11] Godfrey, however, brings nuance to this approach by arguing that thinking about abstraction as art, which "refuses" meaning, is problematic as, "abstract art works can signify; can make meaning in so many different ways" (6). Sasnal's paintings exemplify Godfrey's argument: Sasnal's abstract forms do not deprive the objects of their meaning but are used to create ambiguity, enhance narrative, and activate the audience's readings. Further, Sasnal resorts to these forms specifically when directly referencing the Holocaust.

Figure 14: Gerhard Richter, *Untitled (Grey)*, oil on canvas, 1968.

The way in which the artist uses abstraction alludes to the art of Gerhard Richter, who also works with the Holocaust theme. The point of contact between the two artists is that they both aim to work through their position towards Shoah history. However, they come from different contexts. Richter, a German who has lived through WWII and whose family was involved in Nazism,

[10] The discourse on Holocaust representations has a rich historiography. Adorno, in *Cultural Criticism and Society* (1983), wrote, "to write poetry after Auschwitz is barbaric," (34) setting the framework for looking for means of engagement that do not redeem the crimes by offering aesthetic pleasure (Godfrey 11).

[11] Readers can think about Anselm Kiefer's art that hovers between abstraction and figuration when alluding to the Holocaust history and German post-war identity.

directly relates to this history. Sasnal's case is more ambiguous. He belongs to the generation of postmemory and only "remembers" the Holocaust through existing accounts and images. It is yet evident that both artists resort to a similar abstract language when aiming to depict the Shoah. Godfrey and Redzisz, in their Polin lecture held in conjunction with Sasnal's show, "Uses and Abuses of Abstraction" (2022), visualize the internal links between their art using the works *Shoah (Forest)*, 2003, (see figure 15) and *Untitled (Grey)*, 1968, (see figure 14).

Figure 15: Wilhelm Sasnal, *Shoah (Forest)*, 2003, oil on canvas, Boros Collection, Berlin.

Sasnal's *Shoah (Forest)* shows a still from Claude Lanzmann's film "Shoah" (1985), where the filmmaker, a Holocaust witness, and the translator, Barbara Janicka, are walking through Poland's landscapes. Lanzmann's work is vital for Sasnal as it was one of the first objects that disrupted Poland's uncontested narrative of innocence during WWII. For Sasnal, "Shoah" completely revised his understanding of the scale of the destruction committed by Polish citizens, which he realized was vastly larger than he had thought (Sasnal, "Oprowadzenie"). These forests, which before the war were seen as innocent nature, became a space of execution, resistance, and Nazi crimes (Godfrey, "Uses"). Sasnal strays away from figuration and realism in their representation.

Richter's *Untitled (Grey)* is part of a series of paintings he produced in the 60s and 70s, which were displayed in Tate Modern's exhibition "Gerhard Richter - Panorama" in 2011. While the series was widely understood as a rejection of the photographic image (Godfrey, "Uses"), the exhibition's catalog states that

> "[Richter] suggested another dimension to these works: 'grey monochrome paintings [were] the only way for me to paint concentration camps. It is impossible to paint the misery of life, except maybe in grey, to cover it'" (Richter qtd. in Godfrey, "Gerhard Richter").

Sasnal's and Richter's oil paintings come together in their use of thick, spiraling strokes of paint layered on top of one another. The brushstrokes communicate a certain impossibility of representing these landscapes as they appear in real life. The abstract forms are used to remedy coming in direct contact with this history. In that sense, Richter and Sasnal maintain a strict distance between themselves and the depicted image. Situating Sasnal's use of abstraction in a broader artistic context enriches the current understanding of his creative choices. It helps to view them as part of a more comprehensive artistic response to painting images of the Shoah.

7.3 Narrative Enhancement

Redzisz argues that

> "Sasnal's paintings from the last two decades are strongly rooted in reality. A reality that is faulty, banal, and lacking. Abstraction as a metaphor can stand for this deficiency. But Sasnal's paintings are not allegorical, he employs abstraction as a method of representation, and in his work, it enhances the narrative aspect."

Indeed, as it functions in Sasnal's artworks, abstraction becomes a tool for narrative intensification. When looking at *Shoah (Forest)*, before considering the title, the viewer is not immediately led to believe that they are looking at a forest marked by Holocaust history. Nevertheless, the green spirals suggest one is looking at nature, and the three small figures suggest an exchange is happening. The viewer is thus invited to interpret the forms and is then confronted by the title. Rosa writes about the artwork, arguing that, "imagination can lead us to 'see' a forest, and the artist may merely point the way," which he does through the titling of the work. Rosa explains that, "[r]eferencing the Holocaust, 'Shoah' is a powerful word that immediately provokes the viewer to become alert, and rapid associations are drawn up." Sasnal's titling provokes the viewer to see a forest in the layers of paint and to quickly realize that they are looking at a crime scene. The green spirals become objects of the viewer's projection: one begins piecing the painting's elements together to create the narrative. We now see in the abstract forms a contaminated landscape that cannot be trusted. As put by Redzisz, "[t]he abstraction intensifies [Sasnal's] storytelling by making space for the act of seeing in the

viewer's response." This act of seeing is met by a desire to impose and imagine narratives whose content is subtly led by Sasnal's titling and abstraction.

7.4 A Mobilized Viewer

The narrative enhancement carried out by Sasnal's abstraction is closely linked to another effect created by the artist's method: the mobilization of the audience. Because Sasnal's works leave so many questions unanswered and depictions unspecified, the viewer is left with the immense task of meaning-making. Godfrey further explains this phenomenon: "[i]n front of abstract art works, the lack of a depicted image tends to heighten our awareness of materials, of compositional (or anti-compositional) structures, of the process of looking itself" (*Abstraction*,4). In *Such a Landscape*, the viewer is made hyper-aware of their point of view. The reflective tin walls that line the galleries make it impossible for the viewers to ignore their position within the exhibition as it is constantly reflected at them. When glancing at the artworks that contain Sasnal's signature abstract gestures, the spectator is taken on a journey of seeing. Redzisz outlines this journey in the following way: "[the] visual testimony is hazy, but once the titles are deciphered, and the textual commentary added, the viewer is seduced by the masterful handling of the paint and begins to witness a catastrophe or its aftermath." Cabbage heads emerge as corpses, forests as crime scenes, abstract blocks as Birkenau gates. "To look away when confronting his paintings is ethically impossible, closing your eyes is pointless, the image will chase you (...)" adds Redzisz. The real subject of Sasnal's paintings often remains visually absent, and it is only once we, as viewers, manage to decipher it that we cannot turn away. This process is exemplified in *Kapusta*, where the direct view is the cabbage pile while the real subject is the relationship of the Holocaust victims to Polish culture. The audience is tasked with performing this deciphering process when confronting every painting. The result is an embodied and intense viewing experience that mobilizes the viewer and causes a strong emotional reaction.

7.5 Landscape: A Work of the Mind

Schama writes that,

> "[f]or although we are accustomed to separate nature and human perception into two realms, they are, in fact, indivisible. Before it can ever be a repose for the senses, landscape is the work of the mind. Its scenery is built up as much from strata of memory as from layers of rock" (7).

The landscapes appearing in the Polin exhibition visualize Schama's argument. The display shows the Polish landscape, "treated critically and in an enlarged scope: seen in real life, also in films, photos, texts, poetry, memory, and imagination" (Szabłowski). Sasnal's paintings ask the viewer to activate a critical glance, and to see the narratives, myths, and histories embedded in the seemingly innocent nature. The realization that comes from looking at Sasnal's landscapes is that they are always seen through our memories and associations and thus are inseparable from society and culture. Schama outlines the need for an, "excavation below our conventional sight-level to recover the veins of myth and memory that lie beneath the surface," (Schama 14) and *Such a Landscape* carries out this function. The result is a dissection of how one imagines the landscapes critical to Polish-Jewish history.

7.6 Imagining the Holocaust Landscape

Figure 16: Wilhelm Sasnal, *Untitled*, 2016, oil on canvas.

As argued in the analyses of the previous chapter, there is an ingrained way of thinking about the Holocaust as having no landscape. The logic behind this view is discussed by Maischein, who writes that, "[f]rom a Western point of view, not only the sites of crime but also the sites of memory, seem even today to be located far away" (222). Additionally, the Nazis kept many camps hidden from view. She argues that, "[t]his makes the places of the annihilation of European Jews in Western memory seem like sites without location; they are imagined as unimaginable places" (222). To film "Shoah," Lanzmann traveled to Poland to see what he viewed as "spaces of non-memory" and a "no man's land," and to his surprise, found himself among real Polish towns (Maischein 222). The realization revealed that these were, "concrete places with concrete names" (Maischein 222). Schama writes about his own experience of viewing the Holocaust landscape as "emptied of features" and how, "[it] is shocking, then, to realize that Treblinka, too, belongs to a brilliantly vivid countryside" (26). One can observe a similar reaction in tourists who come to see the remnants of Auschwitz-Birkenau,

located next to the towns of Oświęcim and Brzezinka, where everyday life continues. Sasnal is in conversation with these Holocaust landscape imaginaries, as exemplified by *Untitled* from 2016 (see figure 16).

The painting represents an undefined road that spreads into the distance. The sides are covered in snow with some trees and vegetation. The painting is disturbing: the white and black hazy shading creates an uneasy atmosphere. There is no life present in this scenery, marked by an overwhelming emptiness. Sasnal's painting, read through the exhibition's context, can speak to the vision of the Shoah landscape seen as a 'no man's land'. Schama describes the vision of this land as, "one emptied of (...) color, shrouded in night and fog, [and] blanketed by perpetual winter (...)" (26). Sasnal's painting showcases these features. Yet, this depiction is not grounded in reality but rather in the imagination. The lands of Treblinka and Oświęcim are not empty, nor are they perpetually gray and cold. What is depicted then is not the actual landscape but one that exists inside of the mind. There is an expectation for the post-war landscape to communicate horror, and Sasnal responds to what the audience expects to see.

Moreover, Sasnal's *Untitled* (2016) is also in conversation with the western imaginary of his country's landscape. The public viewed the Holocaust landscape as unimaginable, yet, as witnesses began sharing their accounts and as images of the concentration camps emerged, the imaginary of the Holocaust landscape began to shift. Lanzmann's documentary was crucial in sparking this new perspective. Maischein argues that the outsiders had to "integrate the location in [their] mental symbolic order," (222); the scenes had to be assigned to the crimes. Lanzmann's film achieved this goal by combining shots of the physical places of mass killings with witness statements (Maischein 222).

A new image of the Holocaust landscape had emerged in which Poland became, "a space of memory that seemed to 'speak' about the destruction of the Jews," argues Maischein (222). The process led to the naturalization of terms like "the Polish concentration camps" which the Polish state has been battling through the implementation of memory laws (Belavesau). While there has been a move away from this discursive framework, the western image of Poland remains marked by the WWII tales of deadly territories. Schama, Maischein, and Lanzmann show that this imaginary is characterized by the vision of a cold, relentless landscape that is distant. That is exactly what appears on Sasnal's canvas. He taps into the imagined hostility of the post-war Polish landscape, yet again confronting the viewers with their preconceived assumptions. This time, he explicitly targets the outsider's view of his nation and its implications for thinking about Polish identity.

7.7 A Landscape of Polishness

What is revealed by Sasnal's artworks is that, "[l]andscapes are culture before they are nature; constructs of the imagination projected onto wood and water and rock" (Schama 61). Sasnal opens the door for the audience to project their assumptions and readings onto the nature depicted on the canvas before them. This argument can be enlarged to consider how these landscapes function for Polish identity. Schama argues that,

> "[n]ational identity (...) would lose much of its ferocious enchantment without the mystique of a particular landscape tradition: its topography mapped, elaborated, and enriched as homeland" (15).

Indeed, much of Poland's national identity is heavily intertwined with the Romanticist creation of Poland's heroic landscape, upheld by the literature of cultural heroes such as Mickiewicz[12] or Żeromski [13] (Szymczyk, "Przekleństwo"). When Mickiewicz famously writes, "Lithuania, my homeland," he designates the Polish gentry as the owners of the history that these landscapes symbolize.[14] Jews, although they've inhabited Polish lands for centuries, were seen as a caste rather than as equal neighbors (Schama 29). The heroic image of the Polish landscape is, therefore, discursively constructed to be for and created by Poles. Sasnal and Szymczyk disrupt this smooth imagery by questioning what these lands have seen and what they are a testament to.

"On the eve of the war, Poland was the second largest agglomeration of Jews, after the American Jewry," writes Gross (9). Thus, how can the extermination not be seen as having an immeasurable effect on the everyday lives of Polish society (9)? As argued by Gross, there must be a departure from the view that Jewish-Polish relations were solely, "mediated by outside forces - the Nazis and the Soviets" as it fails to see autonomous interactions between the Jews and Poles (Gross 9). The events of WWII must not be regarded as purely an effect of the two regimes but also of the ingrained antisemitism in Polish society (Gross 123).

Sasnal exposes these issues by interrogating Poland's landscapes. He distorts nationalist narratives that maintain a clear distinction between victim and perpetrator. In looking at his artworks, one must abandon these distinctions and assess their position within this history. Sasnal's art thus shakes the boundaries between passive and active as well as innocent and guilty.

[12] Adam Mickiewicz (1798-1855) was the most prominent Polish Romanticist poet and is remembered as one of Poland's greatest artists. He was also a political figure who fought for Polish independence, which gave him a hero status in Polish culture.

[13] Stefan Żeromski (1864-1925) was a Polish writer and a four-time nominee for the Nobel Prize in Literature. He is known for his passionate writing on the Polish landscape.

[14] He refers to the then Polish-Lithuanian regions as "Lithuania" in a famous excerpt from his 1811 book *Pan Tadeusz*.

He refuses to minimize landscapes to non-intrusive nature. Instead, he emphasizes their capacity to test the intricacies between Polish and Jewish history that have failed to be worked through by Poland. *Such a Landscape* is thus a response to specific imaginaries of the Polish landscape in Polish and non-Polish society. Sasnal questions the perceptions rooted in his country's arts and literature while simultaneously interrogating the post-war foreign view of Poland. Through this, he shows that depictions of nature are never only pure reflections of what is in front of our eyes, but always produce and are products of their specific political, cultural, and societal contexts. The exhibition thus presents a landscape of Polishness and situates Jewish history at its core.

8 Conclusion

This capstone has studied Poland's uneasy relationship towards the strand of its WWII history through Sasnal's *Such a Landscape*. I have situated the exhibition in its spatial and cultural context to show how it adds valuable insight to the discourse of Polish-Jewish relations. The analysis of the specific artworks revealed how the image of the Holocaust landscape rests on the viewer's imaginaries and assumptions and, therefore, is vital to understanding society and culture even though it is manifested through nature. Further, I have shown how Sasnal breaks down the idea that landscapes are innocent by implicating them through symbols of Polishness in the Holocaust history. Herein, Sasnal demonstrates how landscape can be a crucial tool for interrogating the traces of the Holocaust in Poland's everyday life, affirming the inner links between Polish identity and Holocaust history.

This research emphasizes the significant role of the visual arts in addressing Poland's contemporary and historical relationship to the Holocaust. Studies by Tokarska-Bakir, Gross, and Redzisz highlighted the need for Poland to work through its complex war-time history. Sasnal emphasizes that his exhibition comes from this need ("Oprowadzanie"). As a Pole, he had to come to terms with his implication in WWII crimes and critically analyze his position within this history. Sasnal believes Poland should do the same ("Oprowadzanie"). Therefore, a deconstruction of traditional categories of implication is needed, moving beyond victims, perpetrators, and witnesses to include the unexplainable and unregulated public actions.

For there to be space for this rethinking, every Pole should assess their position in Polish-Jewish history, also concerning the WWII crimes. The hate against the 'other' that manifested itself in WWII and post-war antisemitic politics still inscribes itself onto the present Polish landscape. A country that used to house the second largest population of Jews is now devoid of their presence. This absence is a contemporary reminder of the antisemitism that, in different forms and extremes, on top of Nazi crimes, forced the Jews out of the country. And while the prescribed category of the 'other' has been dynamically changing over time in Polish politics – it has been the Jew, the woman, the immigrant, the non-heterosexual – it still manifests itself in Poland's contemporary politics. Therefore, it is urgent to work through Poland's difficult past to gain a richer perspective on what is happening in the present.

This research has shown that the arts prove to be a valid place to look towards when evaluating Polish history and identity. Art history can thus be a locus for alternative readings of the Polish-Jewish relations, one that moves away from standardized and state-imposed narratives. What can further enrich this approach is the inclusion of more voices. As my research focused on interrogating Polish-Jewish history from a non-Jewish Polish artist's perspective, the next step could be to engage with this history from a Jewish perspective. This could add to the already outlined complexity of the relationship and fill the possible gaps left by my research.

Acknowledgements

I would like to thank my supervisor, Dr. Janna Schoenberger, who has provided pivotal support throughout this process. I am grateful for all the art history knowledge that I gained through our cooperation on this project and by taking many of her great courses over the last three years. I also want to extend a special thanks to Dr. Jonathan Gill at the writing center for his invaluable feedback. Additionally, I would like to thank Dr. Marco de Waard for introducing me to looking at art through a Cultural Memory Studies perspective that informed my capstone. Lastly, I would like to acknowledge my reader, Dr. Angela Bartholomew, for participating in this process.

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